12.710 Marine Geology and Geophysics
Structure of the Earth

Stacey, Chapters 4 & 5
Fowler, Chapter 4

Introduction

- Most of our understanding of Earth structure is indirect, using surface observations of seismic waves, gravity, magnetics, electrical conductivity. Today we will review the structure of Earth, with minimal emphasis on lithosphere, because that will be discussed in a separate lecture. The most detailed information on Earth's internal density structure comes from seismic observations.

Compositional Stratification

Earth (radius 6371 km) divided into a **crust, mantle** and **core**.
- crust:
    continents -- ~30 km average thickness with considerable variation; granitic composition [granite is a silica-rich rock composed of quartz [SiO_2], feldspar [(K, Na, Ca)$Al_2Si_2O_8$] and mica (a chain silicate)]
    oceans -- 6 km average thickness with little variation; basaltic composition (basalt is a basic rock composed mostly of pyroxene [(Mg, Fe)SiO_3) and feldspar].
- mantle:
    upper mantle – rich in the mineral olivine $(Mg, Fe)_2SiO_4$; olivine transforms to more densely packed **spinel** ($MgSi_2O_4$) due to pressure increase with depth
    lower mantle -- perovskite (Mg, Fe)SiO_3 and magnetowustite (Mg, Fe)O
- core (radius 3485 km); iron/nickel oxide
    outer core: liquid
    inner core: solid

If temperature increases with depth in the Earth why is the outer core liquid and the inner core solid? Because the melting temperature of an iron-nickel solid solution increases with pressure such that iron-nickle is below the melting temperature in the inner core.

Complications to structure:
- **low velocity zone** - 100-200 km zone of partial melting
- **400-km discontinuity** - olivine spinel structure due to pressure increase with depth.
- **650-km discontinuity** – transformation of spinel to perovskite and a mixture of oxides; very closely packed configuration (MgO, FeO, SiO_2 stishovite). This phase transition is also a consequence of pressure increase with depth.

The compositional stratification within the Earth is a consequence of **melting**. This fact underscores the role of the early thermal state of the Earth in order to understand how Earth's internal structure developed.
- The core formed as a consequence of **differentiation** of (Fe-Ni) metal component from silicate-iron initial composition of Earth during and shortly after accretion.
- Oceanic crust formed by melting of upper mantle; mantle that has been modified compositionalluy due to melting is referred to as **depleted mantle**.
- Continental crust is formed by melting and re-working of oceanic crust (*i.e.* granite is a secondary melt product).

Seismology: Types of Elastic and Seismic Waves

On the relatively short time scales over which seismic waves propagate through the Earth, the constitution of the Earth's interior to first order approximates an **elastic material**. In the simplest case of linear elasticity, stress is linear proportional to strain. The relationship between stress (σ_{ij}) and strain (ε_{ij}) for a linearly elastic material is

$$\sigma_{ij} = \lambda\Delta\delta_{ij} + 2\mu\varepsilon_{ij} \qquad (1)$$

where $\Delta = \Delta V / V_o$ is the dilatation, a measure of the change in volume over initial volume, and μ and λ are **Lame's constants**, properties of the material. The constant μ is the **shear or rigidity modulus** which is a measure of the resistance of the material to shear strain and is a measure of stress needed to change the shape of a material, while λ combines with μ to define the compressibility of a material discussed later. <u>Because the Earth responds elastically over short time periods, seismic waves can to first order be treated as elastic waves.</u>

Seismic waves are well described by solutions of the **wave equation**

$$\frac{\partial^2 u}{\partial t^2} = v\frac{\partial^2 u}{\partial x^2} \qquad (2)$$

where *u* refers to distance, *t* to time and *v* is wave speed. When equation (2) is solved for relevant boundary conditions, it yields solutions that describe the propagation of stress waves in an elastic medium. Such wave solutions provide information on internal structure by yielding elastic parameters that describe the Earth's response to the propagation of the stress waves on time scales of second to minutes to hours. Thus they sample the **short term properties** of the Earth's interior. The **long term** or **viscous** response of the Earth must be determined from other methods that we cover later in the course.

<u>**Body Waves**</u>

Propagate through the Earth's interior. In a homogeneous medium they would consist of spherical wavefronts, but they are refracted and reflected due to layering and lateral heterogeneities. There are two kinds of body waves:
- **P waves** - "primary"; compressional waves; alternating compression and rarefaction of the medium; transmitted by solids, liquids, gases; first seismic arrivals.
- **S waves** - secondary or shear waves; particle motion transverse to direction of propagation; transmitted only by solids (*i.e.* substances with significant shear strength); result in pure shear of medium; secondary seismic arrivals.
    - S waves are **polarized** due to their interactions at horizontal boundaries ; these waves provide additional information on structure.
    - **SV polarization** - vertical
    - **SH polarization** - horizontal

A key aspect of body waves is their **velocity**, which provides information on internal structure. In an isotropic solid, the velocity of a P wave is designated α

$$\alpha = \sqrt{\frac{K + \frac{4}{3}\mu}{\rho}} \qquad (3)$$

and velocity of an S wave is designated β

$$\beta = \sqrt{\frac{\mu}{\rho}} \tag{4}$$

In the above expressions

$$K = \lambda + \frac{4}{3}\mu \tag{5}$$

is the **bulk or compressibility modulus**, which is a measure of stress needed to achieve a fractional change in volume, and ρ is density.

Note that propagation of an acoustic wave in a fluid medium causes compression and rarefaction that is hydrostatic. In this case, the rigidity $\mu = 0$ and this is why there are no S waves in a fluid.

Body waves are non-dispersive and have amplitudes that go as *1/r*, where *r* is distance. They are a primary data set used in the determination of Earth's internal density structure. More on that later.

Surface Waves

Surface waves do not penetrate the Earth's interior, rather most of their energy resides close to the surface. There are two principal kinds of surface waves. In both cases the amplitudes of particle motion decrease exponentially away from the boundaries that guide the waves; amplitude is very small at depths less than a wavelength. The two kinds of surface waves are:

Rayleigh waves - propagate like SV waves with a coupled P wave component.
- particle motion like retrograde ellipses; restoring force provided by elasticity of medium.
- for a range of Poisson's ratio (a material property of elastic material) $v_R \approx 0.92\beta$ for the fundamental mode, so it is similar to that of a shear wave.

Love waves - propagate like SH waves. Do not propagate on the boundary but require velocity layering with increase of shear speed β at depth. Love waves remain confined to the near-surface by repeated reflections in a higher shear speed medium below. Penetration into deeper part of Earth occurs for longer wavelength waves.

- Wavelength (λ) related to period (*p*) and phase velocity (*v*) as $\lambda = pv$
- Wave speeds of surface waves are sensitive to structure to depths of about a third of a wavelength.

Surface waves are useful for finding velocities in low velocity zone because body waves don't bottom out there.

Surface waves are dispersive, and their waveform changes with time.

Free Oscillations

Free oscillations are very long period waves; curvature of Earth influences dispersion.

After a large earthquake the Earth resonates at numerous discrete nodal frequencies, which may be viewed as a standing wave resulting from surface waves traveling in opposite directions. The resonant frequencies are related to the phase speeds of corresponding surface waves. Free oscillations thus give a global average of the same information as the surface waves.

There are two different kinds of free oscillations: **spheroidal** and **toroidal.** As for body waves, the fundamental motions are alternating dilatations and compressions, but unlike body waves free oscillations are spherically shaped. Spheroidal oscillations consist of dilitation and compression only, while the toroidal oscillations also exhibit a torsional or shear component.

Over a thousand free oscillation modes have been identified, and these constitute a data set on internal Earth structure that is independent of body waves.

Seismic Phases

Seismic ray paths are denoted by abbreviations, indicating the kinds of waves and where/how they propagated:

P	mantle P-wave
S	mantle S-wave
p	P-wave reflected from Earth's surface close to earthquake focus
s	S-wave reflected from Earth's surface close to earthquake focus
K	P-wave that propagates through the outer fluid core
I	P-wave that propagates through the inner solid core
J	S-wave that propagates through the inner solid core
c	reflection from the mantle-outer core boundary
i	reflection from the outer core-inner core boundary
LR	Rayleigh wave
LQ	Love wave

Ray Paths

Consider the refraction of seismic waves in a spherical, layered Earth. As in optics, seismic waves bend when they propagate through a medium with a different refractive index and they also undergo changes in wave speed. In the Earth density generally increases with depth so incident wave paths tend to shallow. In the low velocity zone and outer core wave velocity decreases so wave paths in these zones steepen. To quantify the refraction and reflection of waves in the Earth it is convenient to utilize **Snell's Law**, which provides a relationship between the amount of refraction and the velocity of a wave that passes across the boundary of two media that have different refractive properties.
Stacey presents a derivation for Snell's Law by considering refraction of a plane wave at a plane boundary and assuming that the distance traveled by a point on the wave in time Δt is proportional to the wave speed, which differs in different media. Angles i_1 and i_2 are related by triangles *ABC* and *ABD*. Thus

$$\sin i_1 = \frac{BC}{AB} = \frac{v_1 \Delta t}{AB} \text{ and } \sin i_2 = \frac{AD}{AB} = \frac{v_2 \Delta t}{AB}$$

so that

$$\frac{\sin i_1}{\sin i_2} = \frac{v_1}{v_2} \tag{7}$$

This is Snell's Law. Now consider a spherical Earth in which the seismic wave speed varies radially. Assume that each layer has uniform material properties. Variation of material properties can accommodated by adding more layers. According to Snell's Law

$$\begin{aligned} \frac{\sin i_1}{v_1} &= \frac{\sin f_1}{v_2} \\ \frac{\sin i_2}{v_2} &= \frac{\sin f_2}{v_3} \end{aligned} \tag{8}$$

where the i's and f's are angles between the incident rays at layer interfaces and and v's are velocities. It is also convenient to bring in a relationship between refraction angle and the distance *(r_i)* from the center of the Earth. Using similar triangles for the geometry in Fig. 5.11 of *Stacey*

$$q = r_1 \sin f_1 = r_2 \sin i_2 .$$

If we cross-multiply (8) by r_1 and r_2 we see that

$$\frac{r_1 \sin i_1}{v_1} = \frac{r_1 \sin f_1}{v_2} = \frac{r_2 \sin i_2}{v_2} = \frac{r_2 \sin f_2}{v_3}.$$

This form of expression can be extended to any number of boundaries or, in the limit of an infinite number of thin layers, to a layer in which speed increases with depth. A general statement can be written

$$p = \frac{r \sin i}{v} \tag{9}$$

where p is a **ray parameter** that is constant for a given wave. This expression is true for refractions, reflections and conversions of waves to other phases.

Now consider two waves that originate from the same point that have slightly different ray parameters. Their distances correspond to Δ and $\Delta + d\Delta$. Here Δ is the angular distance subtended by surface points on the spherical Earth. If the path difference between the rays is QN then the difference in the travel time *(T)* is

$$dT = \frac{QN}{v_o} = \frac{PQ \sin i_o}{v_o} = \frac{r_o d\Delta \sin i_o}{v_o}$$

where v_o is the wave speed in the surface layer. Thus

$$\frac{dT}{d\Delta} = \frac{r_o \sin i_o}{v_o} = p \tag{10}$$

So the ray parameter p is the slope of the travel time curve (aka travel time as a function of angular distance from the earthquake source). <u>The ray parameter is thus important because it can be directly linked to the total angular distance a wave travels in the Earth</u>. From expression (10) travel time plots can be interpreted in terms of the Earth's seismic velocity structure.

Travel Time Plots

Observations of many P and S waves produce travel time plots for many phases that can be plotted together to give a picture of the seismic velocity distribution within the Earth. This picture exhibits some complications that have represent fundamental aspects of Earth structure.

If seismic wave speeds in the Earth increased progressively with depth, then $p(\Delta)$ would always be continuous. But two effects cause breaks:

(1) an increase in speed w/depth causes rays to bend up or shallow; a decrease in speed with depth causes them to bend downward or steepen; so a **low velocity zone** can produce a range of Δ where there are no travel times. This is called a **shadow zone.**

(2) A sharp increase in speed with depth causes a decrease in the incidence angle of the received wave. An example is the transition zone in the mantle, where phase changes cause density to increase significantly with depth. In this case three direct wave arrivals may be observed. This is known as a **triplication**.

Internal Velocity Structure from Seismic Waves

After travel times of many seismic waves are measured and travel time curves are plotted, the results can be used to determine the variation of seismic velocities in Earth. Consider an element in a wave path ds that makes an angle i to the radius at the point, so the contribution to Δ by the element is $d\theta$. Then

$$r d\theta = \sin i \, ds$$

which can be used to substitute for $\sin i$ in (10) as

$$p = \frac{r \sin i}{v} = \frac{r}{v} \cdot \frac{r d\theta}{ds}.$$

Note that

$$(ds)^2 = (dr)^2 + (r d\theta)^2$$

and so ds can be eliminated from the previous two expressions to give

$$\left(\frac{r^2 d\theta}{vp}\right)^2 = (dr)^2 + (r d\theta)^2.$$

Rewriting, we find

$$\frac{d\theta}{dr} = \frac{vp}{r\left(r^2 - v^2 p^2\right)^{1/2}}.$$

Now define a new parameter $\eta = r/v$ and re-arrange to yield

$$\frac{d\theta}{dr} = \frac{p}{r\left(\eta^2 - p^2\right)^{1/2}}.$$

Next we may integrate with respect to r from the deepest point of penetration of a ray r' to the surface r_o to give **Abel's integral equation**

$$\frac{1}{2}\Delta = \int_{r'}^{r_o} \frac{p\,dr}{r\left(\eta^2 - p^2\right)^{1/2}}. \qquad (11)$$

Note that equation (11) gives η and thus v as a function of r from observed values of p and Δ. The solution of (11) is the well known **Wiechert-Herglotz** formula

$$\int_0^{\Delta_o} \cosh^{-1}\left(\frac{p}{p_1}\right) d\Delta = \pi \ln\left(\frac{r_o}{r_1}\right) \qquad (12)$$

where p_1 is the value of p at $\Delta = \Delta_1$. Given a ray that penetrates to radius r_1 at a wave speed v_1, Equation (12) can be numerically integrated over any interval for which $p(\Delta)$ is continuous, and gives $v(r)$ over this range. Different seismic phases give overlapping ranges that allows interpolation over the breaks. Earth models now include significant amounts of free oscillation data. But during the times before that data type was widely used velocity models were based entirely on body wave travel times and were reasonably close to the currently accepted structure.

Preliminary Reference Earth Model

The **Preliminary Reference Earth Model or PREM** is a seismic reference model of the Earth developed by Dziewonski and Anderson (1981). It was derived from joint inversion of free oscillation periods, mass and moment of inertia, and travel time and distance data from body waves. It is a commonly used basis of comparison for high resolution regional studies from **mantle tomography**.

Internal Density Structure and Elastic Moduli

So now we have a radial velocity structure for the Earth and we now want to derive from this as much information as possible on internal density and mechanical structure. Equations (4) and (5) relate these parameters. HOWEVER, even if we know how velocity varies radially within the Earth, these equations alone cannot give us the density and elastic parameters ρ, μ, and K. The reason is that we have two equations and three unknowns. So we can relate ratios of the three to seismic wavespeeds, such as

$$\frac{K}{\rho} = \alpha^2 - \frac{4}{3}\beta^2$$

and

$$\frac{\mu}{\rho} = \beta^2,$$

but another equation is needed to uniquely determine all three unknowns.

Enter the **Adams-Williamson equation.** For tractability, we make the simplification that density increases with depth only because of self-compression (*i.e.* we ignore phase changes and changes in composition).

We divide the Earth up into many infinitesimal thin spherical shells with uniform physical properties. Consider the increase in pressure δP which results during the descent from radius r to $r+\delta r$

$$\delta P = -g(r)\rho(r)\delta r$$

where $\rho(r)$ is the density of the shell and $g(r)$ is the gravitational attraction. This increase in pressure is due only to the weight of the shell. The relationship between pressure and radius can be rewritten

$$\frac{dP}{dr} = -g(r)\rho(r), \qquad (13)$$

which is just the gradient of the hydrostatic pressure. The above expression has a minus sign because pressure increases as radius decreases. We may write $g(r)$ in terms of the universal constant of gravitation G:

$$g(r) = \frac{GM(r)}{r^2}$$

where $M(r)$ is the mass contained within r. Next, we substitute into (13) to get

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}.$$

We then bring in density:

$$\frac{d\rho}{dr} = \frac{dP}{dr}\frac{d\rho}{dP} = -\frac{GM(r)\rho(r)}{r^2}\frac{d\rho}{dP}. \qquad (14)$$

To find $d\rho/dP$ the bulk modulus K is used. Specifically

$$K = -\frac{dP}{dV/V}, \qquad (15)$$

where the expression corresponds to the change in pressure for a fractional volume *(V)* change. (This is a material constant.) The minus sign is due to the fact that volume decreases as pressure increases. To bring density into the picture we know that

$$\frac{dV}{V} = -\frac{d\rho}{\rho}$$

which, substituting into (15) gives

$$K = \rho \frac{dP}{d\rho}.$$

So equation (14) can be rewritten

$$\frac{d\rho}{dr} = -\frac{GM(r)\rho(r)}{r^2}\frac{\rho(r)}{K}. \tag{16}$$

Now from (4) and (5) we see

$$\frac{d\rho}{dr} = \frac{-GM(r)\rho(r)}{r^2(\alpha^2 - \frac{4}{3}\beta^2)}. \tag{17}$$

Equation (17) represents the mass in a thin spherical shell, so we need to now integrate over all the mass shells. This gives

$$\frac{d\rho}{dr} = -\frac{4\pi G\rho(r)}{r^2\left(\alpha^2 - \frac{4}{3}\beta^2\right)}\int_{r_1=0}^{r_1=r}\rho(r_1)r_1^2 dr_1\,. \tag{18}$$

Equation (18) is the **Adams-Williamson equation** and it **gives density as a function of depth in the Earth for the simple case where density increases with depth due to self-compression**. In fact we know that density also changes due to composition and phase changes so equation (18) is usually used in a piece-wise manner in the mantle and the core. Note that the total mass of the Earth is known and is a constraint on (18).

In practice, there are also corrections added to account for the non-adiabatic temperature gradients, which have an effect on density. Together with seismic velocity models, equation (18) is iteratively adjusted to yield the Earth's internal density structure.

Seismic Tomography

Seismic tomography is a technique to invert many digital seismic waves to yield very detailed velocity models of mantle. Tomography requires a network of digital , long period seismic stations. As part of this method seismologists measure phase and group velocities, as well as waveforms for many earthquakes. These observations are inverted to produce s best-fit 3-D model of velocity structure of upper mantle. Such moels are compared to a standard velocity model like PREM.

Determine fast (cold) and slow (warm) areas.
- Cold: trenches
- Warm: ridges

Provides information on internal density (chemical, thermal) structure.

Velocity Anisotropy

The velocity of seismic waves is greater parallel to the long crystal axis than perpendicular to it. **Velocity anisotropy** is the dependence of seismic velocity on direction. Provides info on mantle flow structure as any flow will cause olivine crystals to align. Shows horizontal flow beneath shields and vertical flow beneath ridges and trenches.