12.710 Marine Geology and Geophysics
Plate Tectonics and Plate Motions

Fowler: Chapter 2 & 8
Kennett: Chapter 5 & 6

Plate Boundaries

Deformation of the surface of the Earth is mostly confined to plate boundaries. This observation has prompted the **rigid plate assumption** in plate motion studies, whereby deformation is assumed to be restricted to plate boundaries. The nature of the **stress field** and style of deformation associated with plate motions is most simply determined from **earthquake focal mechanisms**.

First Motion Analysis

The direction of the initial movement in an earthquake is preserved in seismic waves and can be detected at stations remote from the epicenter. In an infinite, homogeneous medium the pattern of first motion as measured in a P-wave would be simple. The medium could be divided into four quadrants and one pair would experience initial compression (motion away from the source) and the opposite pair would experience dilation (movement toward the source). This pattern represents a **double couple**. The boundaries of the quadrants would include a **fault plane** and an **auxiliary plane** that is normal to the fault plane. In practice, velocity variations within the Earth cause refractions of these **nodal planes**. By correcting for these refractions, first motions from seismograms at widely distributed sites are used to deduce nodal planes and slip directions. These are called **focal mechanisms** or **fault plane solutions.**

Because stresses in the Earth must balance ($\tau_{xy}=\tau_{yx}$), there can be no net couple acting on the medium. The torque must balance the slip. In practice the fault plane cannot be distinguished from the nodal plane solely from the focal mechanism. Additional information (usually geological context) must be invoked to identify the fault plane.

Graphical Representation of focal mechanisms:
The graphical representation is the projection onto a horizontal plane of the compression (by convention black) and dilation (white) quadrants for downgoing waves, *i.e.* P-waves into a hemisphere beneath the source. The resultant **"beachball"** diagrams provide an indication of extensional **normal faults**, compressional **reverse faults**, **strike-slip** faults that result in shearing deformation, or a combination.

Types of Plate Boundaries

- **ridges** – loci of plate spreading; extensional environment; formation of new seafloor
- **trenches** – zones of compression; subduction of old seafloor
- **transform** - narrow zones of shear; plates slide past one another

Note: the surface area of the Earth must be conserved by plate formation, shearing and consumption.

The Wilson Cycle

The plate boundaries are the manifestation of the **Wilson cycle**, named after **J. Tuzo Wilson,** who first sorted out the sequence of events that explained the surface manifestation of plate tectonics. A new ocean forms by **rifting** of a pre-existing continent. **Normal**

faults associated with the **rift valley** are due to extensional stresses in the lithosphere above upwelling mantle convective cells. Continued extension produces a **spreading ridge** at which new seafloor is created. The seafloor cools as it moves away from the spreading ridge and as it does so it becomes denser and less buoyant. Eventually, the oceanic lithosphere becomes sufficiently **negatively buoyant** to **subduct** and produce regions of mantle convective downwelling. This occurs at regions of plate boundary **convergence**. The driving forces for plate tectonics will be discussed in a later lecture.

General Characteristics of Plate Boundaries

Ridges

Mid-ocean ridges are extensional plate boundaries marked by the formation of oceanic crust. Ridges are zones of high heat flow (>10x "normal") but with significant local variability. The seismicity is characterized by shallow earthquakes (<30 km depth). While ridges represent extensional environments, earthquake mechanisms are variable because randomly-oriented thermal stresses dominate the near-ridge stress state.

Spreading ridges are of two general types: slow-spreading and fast-spreading.

	Slow-spreading ridge	Fast-spreading ridge
Example	Mid-Atlantic Ridge (MAR)	East pacific Rise (EPR)
Spreading rate (cm yr^{-1})	~1 to 5	~5 to 10
Morphology	Median rift zone	No median rift zone

The observed differences in axial topography can be modeled by **ductile extension** of a viscous or plastic lithosphere with the added effect of magma accumulation beneath the plate.

- for slow-spreading ridges the width of the median valley scales with the strength of the model lithosphere and the rate of plate cooling or thickening.
- for fast-spreading ridges the topography is controlled by the buoyancy of underlying magma; obviously fast-spreading ridges have access to a significantly greater magma supply than slow-spreading ridges.

There is considerable along-strike variability associated with ridges that has been studied comprehensively. The best explanation for the observations in the context of the evolution of mid-ocean ridges is that extension occurs more or less continuously with intermittent pulses of magmatism.

- for slow-spreading ridges, a broad zone of partial melting occurs in the upper mantle beneath the ridge and a relatively narrow zone of dike injection (as little as a few hundred meters) delivers magma to the surface.
- for fast-spreading ridges, magma chambers have been observed in the crust.

When there is a change in the spreading direction due to a change in the re-orientation of plates (*i.e.* a change in the pole of rotation), the change can be accommodated in two ways:

- asymmetric spreading can occur to cause the ridge to re-orient. Half-spreading rates are different on either side of the ridge and change along strike.
- **propagating rifting** can occur. In this case the ridge jumps to a new orientation. The propagating rift segment lengthens, producing **pseudofaults** which offset the magnetic anomalies that are oriented at oblique angles to the ridge.

These "true-blue" extensional plate boundaries should not be confused with **aseismic ridges** and **crustal plateaus**(*e.g.* Ninetyeast Ridge, Chagos-Laccadive Ridge, Kerguelan Plateau) which are characterized by:

- few earthquakes
- are not associated with boundaries of the tectonic plates
- represent hot spot tracks, microcontinents, and/or uplifted oceanic crust.

Transforms

Transform faults are plate boundaries where rigid plates slide past each other, exhibiting strike-slip motion. Transforms are **conservative** plate boundaries in the sense that crust is neither created nor destroyed. Most transforms occur in oceanic lithosphere but some, most notably the **San Andreas Fault**, are found on the continents.

Transforms are oriented approximately normal to the ridges, or parallel to the spreading direction. The relative velocity along a transform fault is twice the spreading velocity. The relative motion between the adjacent blocks is parallel to the strike of the fault. Active slip only occurs in the zone between offset ridge segments. Outside the active region is a zone of fracturing known as **fracture zones**, which are inactive traces of transforms. This activity is well demonstrated by the distribution of earthquake focal mechanisms. Fault plane solutions show strike-slip motion down to depths of <30 km.

A **leaky transform** exhibits a spreading component; an example is the boundary between Eurasia and Africa, from the Azores triple junction east towards Gibraltar.

Transforms have distinctive morphology. Active transforms are deep linear **transform valleys** <15 km wide, bounded by inward-facing scarps. In contrast to being a single fault displacement is accomplished on many smaller interconnected faults referred to as the **principal transform displacement zone (PTDZ)**. The transform valley is surrounded by a **transform tectonized zone (TTZ)** of intense deformation. At the ridge-transform intersection there is often a deep **nodal basin** thought to be associated with thermally induced viscous head loss ($\Delta P / \rho g$) in the rising asthenosphere beneath the ridge axis due to truncation by colder thicker lithosphere.

Transforms associated with slow-spreading ridges transforms are generally long, narrow and linear. On fast-spreading ridges transforms are less frequent and often extend over broad zones and have numerous ridge-fault segments. On fast spreading ridges **overlapping spreading centers** are sometimes observed. Here a transform does not mark the abrupt termination of both ridge segments but rather the segments overlap slightly.

Oceanic crust beneath transforms is generally thinner than normal crust, sometimes 2-3 km, but the zone of thinning is narrow (<10 km). Layer 3 velocities are sometimes not observed. The process of crustal generation must not be favored in these regions; indeed thermal models by Don Forsyth and others have shown that the juxtaposition of upwelling material against colder transform lithosphere negatively impacts the magma supply.

Adjacent parts of a transform lie at different distances from the ridge crest and so have different ages. Thus the lithosphere thickness varies across the fault zone and the rates of subsidence on each side of a transform differ. Thus transforms have been used to study the thermal structure of oceanic lithosphere.

Ridge-Transform Intersections

Why do divergent plate boundaries break into alternating ridge and transform fault segments? Why don't ridges retain their original, irregular shape which they inherit from original plate breakup? Why do transform faults form perpendicular to ridges? The process of ridge segmentation is not well understood.

It has been suggested that the 50-100 km spacing of transforms on slow-spreading ridges may indicate the horizontal distance over which magma from a single source can supply a ridge segment. In this scenario fast-spreading ridges are not magma limited and indeed transform distributions are more infrequent, hundreds of kilometers apart.

Alternatively, it has been observed that fast-spreading (*e.g.* East Pacific Rise) and intermediate-spreading (*e.g.* SE Indian Rise) ridges are less close to being perpendicular to transform faults than slow spreading ridges (*e.g.* Mid-Atlantic Ridge). It has been proposed that the final, stable configuration taken by ridges and transforms (*i.e.* right angles) is the one that minimizes the mechanical energy associated with the seafloor spreading process. That is, the perpendicular configuration minimizes the length of the ridge crest at the expense of adding however much transform length is needed. When ridge-transform intersections are **oblique** the transforms are shorter than they would be in the normal configuration.

A matter of consideration is the ratio of the energy dissipated along a transform fault to the energy dissipated along a spreading center**.

- At a fast spreading ridge the ratio is ~0.
- At slow spreading ridges the transform and spreading center dissipation have similar magnitudes.

[**Note that the energy dissipated at transforms (largely by earthquakes) is better understood than the dissipation mechanism at ridges, which is quite complex.]

In this scenario the slow-spreading ridges would seem to "work harder" and dissipate more energy, one interpretation of which is that the emplacement of new material at depth is accomplished with more difficulty than at fast-spreading ridges.

Trenches

Ocean trenches or **subduction zones** are convergent plate boundaries where oceanic lithosphere is consumed in the mantle. Subduction occurs in old oceanic lithosphere that has cooled sinks into the mantle due to **negative buoyancy**.

The zone of earthquakes that maps out location of subducting plate is called the **Benioff zone**. Trench-related seismicity is distinctive: At shallow depths earthquake focal mechanisms are mostly extensional and are oriented parallel to Benioff zone. The stresses are associated with bending of the subducting slab. At greater depths and extending down to 700 km earthquakes have compressional focal mechanisms and are aligned parallel to the Benioff zone. These stresses mark resistance to the descending slab.

The surface manifestation of a subduction zone is a **trench** on the oceanic plate that may be as much as 8 km deep and a line of volcanoes in the overriding plate that form due to melting of subducted sediments (and some crust) in the downgoing slab. Magmatic arcs characterized by arcuate shape and are categorized as:

- **island arcs** - where both plates are flanked by oceanic crust, such as Aleutians, Marianas, Izu-Bonin, Tonga (island arc volcanism is similar to that which form continents)
- **continental margin arcs** - where the overriding plate is composed of continental crust, such as western margin of North and South America, Indonesia, and Japan.

Subduction zones often have **back-arc or marginal basins**, between intra-oceanic arcs and continents. These regions are often in a state of extension and spreading may sometimes occur. The trench and **outer rise** of the descending plate are associated with flexural bending of the oceanic plate. Trenches are observed to have a range of dip angles ranging from ~10°-70° and there is variability along strike.

Trenches close to continents may be largely filled with sediments and is called a **fore-arc basin**, behind which lies the volcanic arc. In these cases deformed sediments form an **accretionary wedge**.

Continental margins that have a nearby subduction complex (*e.g.* the Western Pacific) are called **active margins** and those that lack subduction zones (North American Atlantic margin) are **passive margins**.

As the slab descends there are transitions from olivine to the closer-packed spinel structure. This transition to a less dense phase to a more-dense phase releases heat and is an **exothermic reaction**. At lower temperatures in the colder slab the change from olivine to spinel occurs at shallower depths, thus elevating the phase change in comparison to surrounding mantle. The phase change has a density contrast of ~300 kg m-3 and helps drive convection. In contrast the phase change at a depth of 650 km depth from spinel to oxides occurs at a higher pressure than in the surroundings. The thermodynamics is not as well understood as for the shallower phase change but it may be an **endothermic reaction** that absorbs heat. This phase transition thus resists slab penetration but not so much to form a barrier to further penetration. Seismic tomography by Rob van der Hilst and colleagues has shown that slabs descend past the 650-km phase transition, which favors **whole-mantle convection** as opposed to separate convective domains below and above this transition.

Trenches are much less understood than ridges, due to their limited accessibility due to their great depth in the ocean, and the fact that plates subduct and don't leave a detailed record.

The Role of Volcanism

Volcanism is associated with both divergent and convergent plate boundaries. In addition, volcanism may occur in intraplate environments. Almost all volcanoes occur along plate margins.

Near-continuous (in the sense of geologic time) volcanism occurs in association with divergent plate margins, where the **6-km-thick basaltic crust** of the ocean floor is formed. Oceanic crust occurs from melting and passive upwelling of mantle material at mid-ocean ridges. Note that both fast- and slow-spreading ridges produce crust of the same thickness, so the magma supply beneath the fast-spreading ridges must be greater. Also keep in mind that about 1% of oceanic crust is formed at **seamounts**, 1 MY or so off ridge.

Volcanism associated with convergent margins is more spatially irregular along convergent margins and more episodic in time as compared to ridge volcanism. Subduction-related volcanism occurs from melting of mantle mixed with subducted ocean crust and subducted sediments and produces **alkali basalts** that compose the island arcs. **Island arc volcanism** most commonly occurs on the overriding plate where melted sediments from the subducted plate buoyantly rise to the surface. Water in the subducted sediments dictates that island arc volcanism has a high volatile content compared to basaltic ridge volcanics; such volcanism is explosive.

Volcanism also occurs in the interior of plates. A well-known example is the Hawaiian islands, which occur at the end of a chain of islands and seamounts called the **Hawaiian-Emperor Chain** that extends to the Aleutian chain off Alaska in a bilinear distribution. Age-dating of the volcanism shows that the islands grow progressively younger to the southeast, with active volcanism on the eastern-most islands of Hawaii. It is generally believed that the volcanism occurs because the Pacific plate is moving over a **mantle thermal anomaly** that is currently centered under Hawaii. A Pacific plate velocity of 9 cm yr^{-1} explains the linear age progression of the trend. There are other ocean island-seamount trends in the Pacific including the **Marquesas** and **Cook-Austral** islands that exhibit similar trends. However, in some areas there are significant deviations from simple linear age progressions.

There can also be unusual volcanism in association with ridges. The most obvious example is **Iceland**, which has very high rates of volcanism that have produced anomalously thick basaltic crust. Other examples are the Azores and Galapagos islands, where unusual volcanism has produced groups of islands near ocean ridges. These areas of anomalous volcanism were named **hot spots** by J. Tuzo Wilson, and they are the likely surface manifestation of **mantle plumes**. Hot spots can occur beneath both the ocean floor and the continents, and they are not uniformly distributed over the Earth's surface. They have high heat flow and alkali volcanism, are often associated with uplifted basement rocks, sometimes are associated with aseismic ridges, and sometimes are associated with gravity highs. As in the case of Hawaii, some of the hot spots leave tracks with a linear age progression, however, in some places the distribution is random. In some places such as Africa and the western United States, there is a diversity of hot spot volcanism. But the styles are varied for reasons that are not completely understood and include but are not limited to the thermal and compositional environments. Iceland likely represents the very special case of a hot spot that intersects a mid-ocean ridge.

Plate Motions

The first recognition of large-scale plate motions on the Earth came from the visual "fitting together" of African and South America. Global plate motions can now be reconstructed up to 200 Ma in the past, but it is necessary to invoke the **rigid plate hypothesis**. Under this assumption the distance between two points on the same plate will not change. As we will show, this assumption generally holds up well, but the exceptions to the rule are areas of significant tectonic interest.

Start with a Flat Earth:

The relative motions between two blocks can be represented by an angular velocity vector usually denoted as $\vec{\omega}$.

The relative velocities can be determined by vector addition. For example, assume we know the relative velocity of North America relative to Cocos ω_{NA-Co} and the relative velocity of the Pacific plate relative to North America ω_{Pac-NA}. We can then simply find the velocity of the Pacific plate relative to Cocos. This is just:

$$\omega_{Pac-Co} = \omega_{Pac-NA} + \omega_{NA-Co} \qquad (1)$$

In a strict sense this method doesn't work because of internal deformation of the plates. But this can be corrected for.

Rigid Rotations on a Sphere

Thanks to the rigid plate hypothesis the relative motions between two plates can be determined in a straightforward way from **Euler's Theorem**, which states that:

> Any line on the surface of a sphere can be translated to any other position and orientation on the sphere by a **single rotation** about a suitably chosen axis that passes through the center of the sphere. In other words the plate can be rotated to a new position by a **rigid rotation**.

The point where this axis intersects the surface is the **pole of rotation**. Note that ridges and trenches lie on great circles (lines of longitude) from the pole position and transforms lie on small circles (lines of latitude) with centers at the pole of rotation. This was first recognized by **W. Jason Morgan**.

To characterize the rotation in a mathematical sense requires three parameters:
- two coordinates (lat, lon) to locate the pole of rotation
- the angular velocity ω (often given in degrees per Ma)

The combination of these parameters is the **Euler vector.**

The pole of rotation is determined from:
- orientations of ridge crests, magnetic lineations, and transform faults
- angular velocity ω obtained from seafloor spreading velocities determined from the widths of magnetic lineaments, given the requirement that surface area be conserved, or by direct geodetic measurements.

The relative velocity u between plates at any plate boundary is:

$$u = \omega a \sin\Delta \qquad (2)$$

where a is the radius of the Earth and Δ is the angle subtended from the center of the Earth by the rotation pole *(P)* to a point *A* on a plate boundary. Note that the relative velocity is 0 at the rotation pole (where $\Delta = 0°$ and $180°$) and is a maximum $u = \omega a$ at $\Delta = 90°$. Lines of constant velocity are small circles defined by $\Delta = constant$ about the rotation poles. The convention is that a counterclockwise rotation as viewed from outside the Earth is positive. The angle Δ can be related to the latitude θ and east longitude ϕ of *P* and the latitude θ' and east longitude ϕ' of *A* and is given as

$$\cos\Delta = \sin\theta\sin\theta' + \cos\theta\cos\theta'\cos(\phi - \phi') \qquad (3)$$

and the surface distance *(s)* between *A* and *P* is

$$s = a\Delta \qquad (4)$$

where Δ is given in radians.

For a rigid plate model, the sum of relative instantaneous rotation vectors between plates crossed by any circuit on Earth equals 0. This is the concept of **closure** that is used as a test of the goodness of fit.

Methods to determine the directions of relative motion of plates:

- A local estimate of the direction of relative motion between two plates can be made from the **strike of active transform faults**, which form small circles about the rotation pole. So the rotation pole must lie somewhere on the great circle which is normal to the small circle. So if normals are determined for two or more transform faults their intersection corresponds to the position of the rotation pole.

- **Spreading rates** at ridges vary as $sin\Delta$. So if the spreading rate at various locations along the ridge can be determined from the spacing of magnetic anomalies then the rotation pole and angular velocity can be determined.

- **Fault plane solutions** or earthquake **focal mechanisms**.

- **Fault displacements** on land.

- **Satellite geodetic methods**.

More on this later.

Plate Reconstructions

The reconstructions of past plate configurations use many types of "historical" data:

- continental margins (*e.g.* the 2000-m bathymetric contour)
- paleomagnetic data
- paleoclimatic data
- paleobiological data

From paleomagnetic observations we determine the **magnetic declination and inclination of the paleopole**. Note that the **declination gives the direction of the magnetic pole** and the **inclination yields the paleolatitude. BUT, neither of these gives the paeolongitude.** Hence there are uncertainties associated with plate reconstructions.

The usual approach is to plot rotation poles for continents using **coeval** rocks, when available, and to see if the polar wander curves match. If they do, then the continents were merged, and if not, then they were separate.

North America - Europe: the polar wander curves are appreciably offset. But when the two continents are rotated ~40°, the polar wander paths become approximately congruent. This corresponds well to the 38° rotation required to close the Atlantic Ocean and get the best geometrical fit from the 500-m submarine contour. In fact we know that there were three orogenies associated with opening of the Atlantic:

- **Taconic** - mid-late Ordivician 450 Ma; closure of proto-Atlantic (Iapetus Ocean; seen along entire length of Appalachian)
- **Acadian** - lower-Devonian - 385-400 Ma; predominantly a Northern Appalachian event
- **Alleghenian** - Upper Carboniferous and/or Permian - 230-260 Ma; primarily manifest in the southern Appalachians.

These events are known based on the stratigraphic record on the continents.

Then there is the problem of not being able to use seafloor magnetic anomalies for ages > 200 MY because there is no seafloor older than that. Plate constructions for times prior to that are extremely shaky. Paleozoic and older plate reconstructions suffer accordingly because they are limited to using data from the continents. Continental reconstructions can be taken back earlier than 200 Ma using paleomagnetic data but the uncertainties are large.

There have been some particularly interesting paleo-mergers and -breakups:

- From the late Carboniferous to the early Permian (~250 Ma), all the continents were merged into one continent called **Pangea** (a Greek word meaning "all lands").

- At 170 Ma, Pangea rifted to form the northern continent **Laurasia** (NA, Europe, Asia) and the southern continent **Gondwannaland** (SAm, Af, India, Ant, Aust). Between Laurasia and Godnwanna land was the **Tethys** ocean. The Mediterranean, Caspian and Black seas are the only remaining vestiges of the Tethys.

Continental flood basalts – these massive basalt flows are useful in timing continental breakup (*e.g.* Palisades sills at 190-200 Ma Upper Triassic/Lower Jurassic date the last opening of the Atlantic).

Plate Kinematics -- Instantaneous Plate Motions

We can characterize the **kinematics** of plate motion by constructing a global, (hopefully) self-consistent model of the **instantaneous rotations of the Earth's rigid plates**, which must **simultaneously satisfy all available relative motion data**. Kinematics describes the sense of motions but contains no physics *per se* and so does not address the question of dynamics. However, a good kinematic description of plate motion can be (and has been) used to constrain models of mantle dynamics.

On a sphere, the instantaneous motion of a plate will be described fully by an axial rotation around a pole of rotation. The velocity field of a system of plates can be determined by specifying the instantaneous angular velocity vectors of the plates, referred to some fixed frame (usually taken to be the **hot spot reference frame**) with origin at the center of the sphere.

In 1963 and 1965, **J. Tuzo Wilson** suggested that **linear island chains and aseismic ridges** are the surface expression of zones of upwelling in the mantle. He further suggested that the hot spots are fixed with respect to one another, perhaps because they are "anchored" in the lower mantle. If this is the case these features would constitute an **absolute reference frame.** The **hot spot reference frame** utilizes traces represented by the **island chains** and **aseismic ridges**. If hot spots form a fixed reference frame, *i.e.* that of the mantle, then the direction of a hot spot trace on a given plate away from the hot spot should equal the azimuth of the instantaneous motion of the plate in that reference frame.

The Forward Problem

Following the formalism of *Minster and Jordan* [1974] we may assume that any position vector $\bar{r}$ on the surface of a sphere can be expressed in spherical coordinates $\bar{r} = (a, \lambda, \mu)$, where a is radius, λ is latitude and μ is longitude measured positive east. With respect to the reference frame the instantaneous velocity of a point $\bar{r}$ belonging to plate P_i is

$$\bar{V}(\bar{r}) = \bar{\Omega}_i \times \bar{r} \qquad (5)$$

where $\bar{\Omega}_i = (\omega_i, \theta_i, \phi_i)$ is the angular velocity vector of plate P_i, θ_i and ϕ_i are the latitude and longitude of the rotation pole for the i^{th} plate. The rotation rate ω_i is by convention positive for a right-handed coordinate system. If $\bar{r}$ is on the boundary between plates P_i and P_j then the velocity of P_j with respect to P_i at $\bar{r}$ is

$$\bar{V}_{ji}(\bar{r}) = \left(\bar{\Omega}_j - \bar{\Omega}_i\right) \times \bar{r} \qquad (6)$$

where $\bar{V}_{ji} = -\bar{V}_{ij}$. Note that the relative velocity is always taken to be tangent to the plate. Equation (6) may be evaluated to find the latitudinal and longitudinal components of motion

$$V_{ji}^{\lambda} = a\omega_j \cos\theta_j \sin(\mu - \phi_j) - a\omega_i \cos\theta_i \sin(\mu - \phi_i) \tag{7}$$

$$V_{ji}^{\mu} = a\omega_j\left[\cos\lambda \sin\theta_j - \sin\lambda \cos\theta_j \cos(\mu - \phi_j)\right] - a\omega_i\left[\cos\lambda \sin\theta_i - \sin\lambda \cos\theta_i \cos(\mu - \phi_j)\right]$$

which derives from

$$\bar{r} = \begin{pmatrix} a\cos\lambda\cos\mu \\ a\cos\lambda\sin\mu \\ a\sin\mu \end{pmatrix} \tag{8}$$

and

$$\overline{\Omega}_i = \begin{pmatrix} \omega_i \cos\theta_i \cos\phi_i \\ \omega_i \cos\theta_i \sin\phi_i \\ \omega_i \sin\theta_i \end{pmatrix}. \tag{9}$$

We may then write the Cartesian components of $\overline{V}_{ji}$, which are

$$V_x = a\left[\sin\lambda\left(\omega_j \cos\theta_j \sin\phi_j - \omega_i \cos\theta_i \sin\phi_j\right) - \cos\lambda \sin\mu\left(\omega_j \cos\theta_j - \omega_i \cos\theta_i \sin\phi_j\right)\right]$$

$$V_y = a\left[\sin\lambda\cos\mu\left(\omega_j \sin\theta_j - \omega_i \sin\theta_i\right) - \sin\lambda\left(\omega_j \cos\theta_{jj} \cos\phi_j - \omega_i \cos\theta_i \cos\phi_j\right)\right] \tag{10}$$

$$V_z = a\cos\lambda\left[\omega_j \cos\theta_j \sin\left(\mu - \phi_j\right) - \omega_i \cos\theta_i \sin\left(\mu - \phi_i\right)\right].$$

What is really wanted are the latitudinal and longitudinal components V_{ji}^{λ} and V_{ji}^{μ}, that are obtained by taking the inner product of $\overline{V}_{ji}$ with unit tangents to the meridian and parallel to $\bar{r}$. We find

$$V_{ji}^{\mu} = -\sin\lambda\cos\mu V_x - \sin\lambda\sin\mu V_y + \cos\lambda V_z \tag{11}$$

$$V_{ji}^{\lambda} = -\sin\mu V_x + \cos\mu V_y$$

And so the components of equation (5) are now specified.

As posed, this is a **forward problem**. Given the three parameters $\overline{\Omega}_i = (\omega_i, \theta_i, \phi_i)$ where *i*=1, *n* where *n* is the number of plates, it is possible to compute the components of relative velocity between two plates at any point along their common boundary.

Inverse problem

Given observations of components of relative velocity at plate boundaries, we may alternatively ask what is the best representation of instantaneous motion? To answer this question we observe $\overline{V}_{ji}$ and the direction of relative motion given by

$$V_{ji} = \left\| V_{ji} \right\| = \left[\left(V_{ji}^{\lambda} \right)^2 + \left(V_{ji}^{\mu} \right)^2 \right]^{1/2} \tag{12}$$

$$\zeta_{ji} \arctan\left(\frac{V_{ji}^{\lambda}}{V_{ji}^{\mu}} \right),$$

and perform a least squares minimization over

$$\sum_{nplates} \left(\frac{\Omega_{obs}^{i} - \Omega_{pred}^{i}}{\sigma_i} \right)^2 \tag{13}$$

where σ_i is the standard error. On a sphere errors follow a **Fisherian distribution**, which goes as $exp(\kappa cos\beta)$, where κ is the dispersion and β is the angular error. A Fisherian distribution is analogous to a Gaussian distribution on Euclidean geometry, and thus implies a **normal distribution of errors**. In practice, a **maximum likelihood approach** is utilized in which poorly-determined data are weighted less than better known data. Such models iteratively improve on a selected starting model until a convergence criterion is met.

What kind of observations go into an instantaneous plate model?

1.) **Relative velocities** - Obtained from spreading rates at mid-ocean ridges determined from **magnetic anomalies** or **direct geodetic measurements**, discussed later.
2.) Directions of relative motions between plates as derived from **transform fault azimuths**.
3.) **Earthquake slip vectors** - Use horizontal projections of earthquake slip vectors to calculate poles of rotation.

Flowchart for an Instantaneous Plate Motion Model

From a magnetic or bathymetric map or satellite altimetry data: estimate spreading rates, transform fault azimuths and associated errors.

⇓

Compute slip vectors from published earthquake focal mechanisms. Estimate errors.

⇓

Examine plate motion data along a single plate boundary, find the best-fit angular velocity vectors, and compare to prior results.

⇓

Analyze closure about local plate circuits by inverting the circuits of three or more plates.

⇓

Simultaneously invert all data to find Euler vectors that have the best fit in a least squares sense. The successful model will test the extent to which plates are rigid

and the extent to which hot spots are fixed. Commonly used instantaneous plate models are the **NUVEL** models of *DeMets et al.* [1990, etc.].

Lack of closure is most often due to internal deformation of a plate. The largest deviation occurs in **Central Indian Ocean**, an **intraplate** deformation zone associated with the collision of the Indian sub-continent into Eurasia. Other zones of intraplate deformation include an area in the Atlantic east of the Azores, and another area of the Atlantic off the northeast coast of South America.

Comparison of Present-Day Plate Motions to Plate Motions Kinematic Models

Space Geodetic Methods for measuring present plate motions:
- **VLBI** - **Very Long Baseline Interferometry** - receive radio signals from quasars on Earth at different locations and triangulate to get the distance between the sites.
- **SLR** - **Satellite Laser Ranging** - range from Earth surface to orbiting satellite with retro-reflectors and determine absolute position.
- **GPS** - **Global Positioning System** - detemine position based on reception on ground of carrier signal from satellites.

There is **excellent agreement** between present-day motions and NUVEL-1 and -2 [*DeMets et al.,* 1990] and RM-2 [*Minster and Jordan,* 1978] kinematic plate models. This demonstrates that the **present-day plate motions are very similar to the motions over the past 10 Ma**. The coincidence of present plate motions and plate kinematic models for past 10 Ma was accomplished by NASA's **Crustal Dynamics Project**.

Plate Boundary Kinematics

Triple junctions and geometrical constraints on plate motions

A plate boundary can end only by intersecting another plate boundary, thus "running into" another plate. The intersection is called a **triple junction**. There are three types of plate boundaries (ridges, trenches and transform faults), and there are in principle 16 possible types of triple junctions. But most of these are not stable configurations. A triple junction is **stable** when the relative motions of the three plates and the azimuth of their boundaries are such that the configuration of the junction does not change with time. But some of the possible configurations are "unstable" and exist only instantaneously before evolving to another configuration. If four or more plates meet at one point the system is always unstable and will evolve into two or more triple junctions.

Of the possible kinds of triple junctions there is one, the **ridge-ridge-ridge (RRR)** triple junction, that is always stable as long as oblique spreading is not occurring. Two are always unstable: one with three transform faults **fault-fault-fault (FFF)** (a transform is represented as "fault"), and **fault-ridge-ridge (FRR)**. The other 13 are stable under some conditions.

The **necessary condition for the existence of a triple junction is that the three vector velocities that define relative motions between plate pairs must form a closed triangle.** Mathematically this criterion is expressed

$$\mathbf{u}_{BA} + \mathbf{u}_{CB} + \mathbf{u}_{AC} = 0. \qquad (14)$$

Quite often, satisfying this condition requires a specific orientation of the boundaries. Note that equation (14) uses relative velocities rather than angular velocities. This is valid in the vicinity of the triple junctions, where the relative velocities are constant.

It is possible to determine the stability (or not) of a given triple junction by drawing a relative velocity triangle and the azimuths of the plate boundaries. Keep in mind that plate motion is not always normal or parallel to the plates.

Consider a **ridge-ridge-ridge (RRR)** triple junction. If we know the spreading rate and azimuths of two of the three ridge segments we can calculate these parameters for the third under the requirement of closure. Assume we have three plates *A*, *B*, and *C* with azimuths α, β, and γ. The spreading velocity of plate *A* with respect to *B* is u_{AB}, etc. Say we know the spreading rate and azimuth of *A* with respect to *B* and *B* with respect to *C*. We want to find *C* with respect to *A*. To find the velocity vector we may use the law of cosines. For a triangle with angles $A+B+C=\pi$, and sides *a, b, c,*

$$c^2 = a^2 + b^2 - 2ab\cos C \tag{15}$$

we have

$$u_{AC} = \left[u_{AB}^2 + u_{BC}^2 - 2u_{AB}u_{BC}\cos(\pi - \beta)\right]^{1/2} \tag{16}$$

and to find the angle γ we use the law of sines

$$\sin(\gamma - \pi) = \frac{u_{BC}}{u_{AC}}\sin(\pi - \beta). \tag{17}$$

In principle, it is also possible to have a **trench-trench-trench (TTT)** triple junction. Such a configuration usually isn't stable, but it can exist for a short period of time. Similarly, a **ridge-trench-transform fault (RTF)** boundary usually isn't stable. The triple junction will evolve by migration of one of the segments to assure continued closure.

To examine the stability of a triple junction, draw a relative velocity triangle and include the azimuths of the plate boundaries. Make the lengths of the triangle proportional to the relative velocities. Draw lines which have the strike of the boundary and pass through the appropriate vertex of the velocity triangle. Extend lines for all three boundaries. If the lines meet at a point the triple junction is stable. If not the geometry will evolve.