12.710 Marine Geology and Geophysics
Thermal & Mechanical Structure of the Lithosphere

Kennett: Chapter 7 (Structure of Oceanic Crust)
Turcotte and Schubert: Chapter 3 (Mechanical Structure of Lithosphere)
Turcotte and Schubert: Chapter 4 (Thermal Structure of Lithosphere)

Shallow Interior Structure of the Earth: Textbook Definitions

Lithosphere (0-125 km)
Asthenosphere (125-700 km)
Mesosphere (700 -2900 km)

The principal dynamical motions in the Earth are a consequence of the manner in which the planet loses heat. The paradigm of plate tectonics is manifest by motions of rigid plates at Earth's surface that trace the motions of mantle convection cells that transport heat to the surface from the deep interior. The outer shell of the Earth is broken into about ten major plates that move as rigid entities and deform (for the most part) only along their boundaries. The Earth's rigid outer shell is termed the **lithosphere**.

The **lithosphere** is most commonly defined on the basis of its mechanical properties. It is the rigid or mechanically strong outer layer of the Earth that can support both horizontal and vertical shear stresses. The lithosphere overlies the **asthenosphere**, a low viscosity zone characterized by low seismic shear wave velocities and high electrical conductivity. Because the asthenosphere is partially (~1% - 10%) molten, it forms a "lubricating channel" that facilitates the motions of the overlying lithospheric plates. The asthenosphere overlies the **mesosphere**, which is the part of the mantle that convects.

In terms of mechanical rigidity: lithosphere>mesosphere>asthenosphere

NEVER, EVER confuse the lithosphere with the crust. The **crust** is a compositional entity, defined on the basis of rock chemistry and mineralogy. **Oceanic crust** consists of **basalts** created at spreading centers by passive emplacement of basaltic (or tholeiitic) magma. Basalts are primarily composed of plagioclase feldspar [$CaAl_2Si_2O_8$, $NaAlSi_3O_8$] and pyroxene [(Mg, Fe, Ca) Si_2O_6], where the magnesium-rich endmember of the solid solution is known as **enstatite**, the iron-rich endmember is orthoferrosilite, and the calcium-rich endmember is **diopsite**.

Magma forms by decompression melting of an underlying mantle, and consists mainly of two kinds of the general rock type **peridotites**, which are ultrabasic rocks that are rich in olivine [$(Mg, Fe)_2$ Si_2O_4]. As you recall from the chapter on Earth structure, olivine is the most common mineral in the upper mant.

- Depleted mantle material (*i.e.* that which has undergone melting) is termed **harzburgite**, which consists of olivine, etstatite pyroxene, and minor amounts of spinel. The parent mantle rock of a harzburgite is termed **pyrolite**.
- Undepleted (read: unmelted) mantle is termed **lherzolite,** which consists of olivine, enstatite and diopsite with minor amounts of spinel or garnet.
- The final member of the peridotite family is **dunite**, which is composed of olivine with minor amounts of enstatite and spinel.

Oceanic crust has a thickness of ~5-20 km, with a typical value being 6 km. Because oceanic lithosphere that forms at mid-ocean ridges cools with increasing age, it eventually becomes denser than the underlying mantle. This "negative buoyancy" causes oceanic plates to subduct into the mantle; hence the term "plate recycling". Because of the subduction process the oldest oceanic crust is much less than the age of the Earth. All oceanic crust is ≤200 Ma.

The oceanic crust can be decomposed into several vertical sections which were originally identified on the basis of distinctive seismic velocities but are now classified on the basis of composition.

> **Layer 1** is composed of sediment and its thickness is variable, generally increasing with distance away from mid-ocean ridge crests.
>
> **Layer 2** is composed mainly of basalt and represents pillow lavas (layer 2A) and sheeted dikes (layer 2B).
>
> **Layer 3** composes the rest of the crust is consists of **gabbro**, a coarse-grained basalt generally formed by intrusion rather than extrusion on the surface. These rocks are sometimes called **cumulates** to reflect the fact that they formed near the base of a shallow subsurface magma chamber.

In some areas sections of oceanic crust and the uppermost mantle have been obducted and are preserved at the surface. These are referred to as **ophiolites**.

Continental crust is derived from partial melting of mantle that overlies subducted oceanic lithosphere to form **island arcs**. Because continental crust has had an extensive melting history, it generally contains relatively light granitic or gneissic (metamorphosed granite) rocks and is more buoyant than oceanic crust. It is sufficiently lighter than mantle that it never subducts and instead tends to "pile up" in plate collisions. Thus continental crust tends to be thicker (20-70 km) than oceanic crust. Another consequence of continental crust not subducting is that it is generally older than oceanic crust; samples as old as 4000 Ma have been identified. As a general rule continental crust has been around for a long time and has undergone a complex history that includes subduction recycling, surficial erosion and hydrothermal differentiation. All of these processes concentrate heat producing elements (^{238}U, ^{235}U, ^{40}K and ^{232}Th) , as well as other **large ion lithophile** elements, into the upper crust. Thus heat production is an important aspect of continental crust, and in thermal models it is essential to account for radioactive heat production, not just cooling.

The lithosphere usually consists of the crust and some part of the upper mantle. In instances of extreme thermal perturbations, the lithosphere may only consist of the upper crust.

How does the lithosphere fit into the "big picture" of plate tectonics?

For geophysicists, who make observations from a vantage point at the surface, the interior of the Earth can be considered a black box. The response of the Earth (*e.g.* earthquakes) to stimuli (stresses associated with plate motions driven by mantle convection) is observed in a spatial and temporal sense. Then the observations are used to constrain the physical and chemical properties of the black box. They are also interpreted in terms of the nature of the stimulus (dynamics). Typically the problem is underdetermined and many kinds of observables must be combined to begin to derive a picture of the structure and dynamics. The solutions are often non-unique and the goal is to eliminate non-uniqueness.

A critical observable is the structure of the lithosphere because it provides an indication of mechanical properties and temperature, which are important for understanding Earth dynamics. If temperature effects can be understood, mechanical properties can in some cases be interpreted in terms of composition.

<u>Definitions of Lithosphere</u>

In practice, various definitions of the lithosphere have arisen from several types of geophysical observables that operate on different timescales. Defining the lithosphere in numerous ways may appear confusing at the outset. However, these different representations provide significant complementary information about the structure and dynamics of Earth's interior.

• **Thermal lithosphere** - In this case the lithosphere is defined as the **thermal boundary layer** (TBL) between the surface of the Earth and the asthenosphere. The thermal boundary layer cools by conduction whereas the region below the TBL cools by convection. The lithosphere/asthenosphere boundary corresponds to the **solidus** (temperature of first melting) of mantle peridotites. The thickness of the oceanic TBL has been estimated using observed subsidence and heat flow as a function of crustal age, determined from seafloor magnetic anomalies. In contrast, it is significantly more difficult to estimate the TBL beneath the continents. The depth to the 1200°C-1400°C isotherm beneath continents in many places is at a depth of 200-250 km, but there are numerous exceptions. In addition, the continents contain more lateral variability in composition and it can be a challenge to separate out effects of temperature from composition in geophysical data.

• **Seismic lithosphere** - is defined on the basis of a drop in seismic velocity. The seismic lithosphere represents the part of the Earth's outer shell that exhibits strength or rigidity on timescales of seconds to minutes (much shorter timescales than the thermal and mechanical structures. Since seismic velocity and attenuation are sensitive to temperature and melt content, the base of the lithosphere can be mapped using variations in longitudinal (P) and shear (S) waves. The lithosphere/asthenosphere boundary is marked by an abrupt decrease in shear wave velocity. Seismic velocities show the low velocity zone (LVZ) or asthenosphere to be ~200-300 km thick. Beneath ridge crests the LVZ comes close to the surface while for old oceanic lithosphere it occurs at depths of 100-120 km. It has proven difficult o recognize the LVZ beneath continental cratons.

How deep the lithosphere extends under the continents has been and continues to be an active matter of debate. Tom Jordan, formerly of MIT, argues for a deep **tectosphere** that extends to >200 km. Others would have it much shallower. The basic problem is a disagreement as to how much seismic velocity changes are attributable to thermal vs. compositional effects beneath continental cratons.

• Sometimes the term **seismogenic lithosphere** is used and this is <u>not</u> the same thing as the seismic lithosphere. The seismogenic lithosphere represents the layer(s) that generate(s) earthquakes within intraplate environments. (This is opposed to the significant magnitude and frequency of earthquakes that mark plate boundaries, that are termed interplate earthquakes.) The seismogenic lithosphere represents those zones of the crust and mantle that are sufficiently cool to rupture by brittle fracture rather than plastic or creep deformation.

• **Mechanical lithosphere** - is a measure of the ability of the strong part of the crust/upper mantle to support significant loads or stresses (of order 10^2 MPa) over geological time scales (of order millions of years). [Note that the unit of stress is the Pascal which has units kg m^{-1} s^{-2}; 1MPa= 10^6 Pa= 10 kilobars). The mechanical lithosphere is often described as an **elastic plate.** As noted previously in our discussion of Earth structure, elasticity is a convenient rheological approximation because strain varies linearly with stress ($\varepsilon \propto \sigma$). Deformation of an elastic material is non-permanent in the sense that it returns to its original shape when the stress is removed. There is no time dependence for an elastic material so it is useful in treating instantaneous deformation or the average deformation over a period of time. In practice, once the failure stress of an elastic material is reached it undergoes permanent deformation and can no longer be treated as elastic.

The thickness of the mechanical lithosphere is found from Earth's response to surface loads such as seamounts, mountain belts, sediments, etc. or subsurface loads such as intrusions. The mechanical lithosphere is often represented as the **effective elastic plate thickness**, *h*, where the term effective is used because the thickness is taken to represent average material properties. The parameter *h* is determined from the equation for flexure of a thin elastic plate applied to the most general case of loading of oceanic lithosphere. Flexure is a widely used approach toward understanding the structure of oceanic lithosphere in an average sense, so it is worth spending some time on the concept. This discussion closely follows that in Turcotte and Schubert's *Geodynamics* text, which has an excellent treatment of flexure at an introductory level.

Consider the case of two-dimensional flexure of an elastic plate. The deflection of the plate can be determined by requiring it to be in equilibrium with respect to the forces and torques exerted on it. We can balance the forces and torques on a small section of the plate between x and $x+dx$. A downward force per unit area $q(x)$ is imposed as a vertical load. Thus the downward force per unit length in the z direction, between x and $x+dx$ is $q(x)dx$. A net shear force V per unit length in the z-direction acts on a cross-section of the plate normal to the plane of the figure that is the resultant of all shear stresses integrated over that cross-sectional area of the plate. A horizontal force, P, per unit length in the z-direction is applied to the plate. The net bending moment, M, per unit length in the z-direction, acting on a cross section of the plate is the integrated effect of all the torques exerted by the normal stresses σ_{xx}, which is known as the fiber stress, on the cross section. Note that z is positive downward.

At a location x along the plate the bending moment is M, the shear force is V, and the vertical deflection is w. At a location $x+dx$ along the plate the bending moment is $M+dM$, the shear force is $V+dV$, and the vertical deflection is $w+dw$. Because this is a two-dimensional problem, the parameters V, M, and P are per unit length in the z direction.

We perform a force balance in the z direction on the small part of the plate between x and $x+dx$ to yield

$$q(x)dx + dV = 0$$

or

$$\frac{dV}{dx} = -q(x). \tag{1}$$

The horizontal forces P exert a net counterclockwise torque $-Pdw$ on the element through moment arm $-dw$. Next we balance the torques

$$dM - Pdw = Vdx$$

or

$$\frac{dM}{dx} = V + P\frac{dw}{dx}. \tag{2}$$

To reduce the number of variables in the problem we eliminate the shear force V from equation (2) by differentiating with respect to x and substituting from equation (1) to yield

$$\frac{d^2M}{dx^2} = -q(x) + P\frac{d^2w}{dx^2}. \qquad (3)$$

Next we wish to express equation (3) in terms of the deflection *w* but to do this we need to relate the moment *M* to *w*. This is facilitated by consideration of the **fiber stress**, σ_{xx}. If the plate is deflected downward σ_{xx} is compressional in the upper half of the plate and tensional in the lower part of the plate. The fiber stress vanishes at the centerline and is referred to as the **neutral surface**. The force on an element of the plate's cross section is $\sigma_{xx}zdz$. We integrate the torque over the cross section of the plate

$$M = \int_{-h/2}^{h/2} \sigma_{xx} z dz, \qquad (4)$$

where *h* is the thickness of the plate.

Of course, σ_{xx} is accompanied by a strain ε_{xx}. Because the *plate is thin* we can assume that σ_{zz} is 0 and that we have a case of **plane stress**. From Hooke's Law we relate stress to strain:

$$\varepsilon_{xx} = \frac{1}{E}(\sigma_{xx} - \upsilon\sigma_{zz})$$
$$\varepsilon_{zz} = \frac{1}{E}(\sigma_{zz} - \upsilon\sigma_{xx})$$

where *E* is Young's modulus and υ is Poisson's ratio, which are properties of an elastic material. A consequence of the thin plate approximation is that $\varepsilon_{zz} = 0$. So one can combine the two expressions above to find

$$\sigma_{xx} = \frac{E}{(1-\upsilon^2)}\varepsilon_{xx}, \qquad (5)$$

and using equation (5) we can rewrite the moment in equation (4) as

$$M = \frac{E}{(1-\upsilon^2)}\int_{-h/2}^{h/2} \varepsilon_{xx} z dz. \qquad (6)$$

Ultimately, we want to eliminate *M* and put the expression in terms of displacement *w*, which requires that we develop a relationship between strain and displacement. We estimate ε_{xx} in the thin plate limit, which requires that plane sections of the plate remain plane. This quantity can be determined from the local radius of curvature of the plate *(R)* and the distance from the midplane*, z*. Assume that the length of the midplane is *l*, and because it is unstrained the length does not change. Elsewhere the length of the plate changes by *Δl*. The change in length *Δl* at distance z from the midplane is

$$\Delta l = -z\phi = -z\frac{l}{R}$$

where the minus sign comes from the fact that positive z indictes contraction. The fiber strain can be written

$$\varepsilon_{xx} = -\frac{\Delta l}{l} = \frac{z}{R}.$$

Now the local radius curvature can be estimates from the change in horizontal slope of the midplane. If w is small, this angle α=-dw/dx. The angle corresponding to the arclength of the plate section f is just equal to the change in a such that

$$\phi = d\alpha = \frac{d\alpha}{dx}dx = \frac{d}{dx}\left(-\frac{dw}{dx}\right)dx = -\frac{d^2w}{dx^2}.$$

So, from above the fiber strain is

$$\varepsilon_{xx} = -\frac{\Delta l}{l} = \frac{z}{R} = -z\frac{d^2w}{dx^2}. \tag{7}$$

Substituting equation (7) in equation (6), the bending moment can be written

$$M = -\frac{E}{\left(1-\upsilon^2\right)}\frac{d^2w}{dx^2}\int_{-h/2}^{h/2} z^2 dz = -\frac{E}{\left(1-\upsilon^2\right)}\frac{d^2w}{dx^2}\left[\frac{z^3}{3}\right]_{-h/2}^{h/2}$$

or

$$M = -\frac{Eh^3}{12\left(1-\upsilon^2\right)}\frac{d^2w}{dx^2}. \tag{8}$$

The coefficient of d^2w/dx^2 in equation (8) is called the **flexural rigidity** D of the plate

$$D = \frac{Eh^3}{12\left(1-\upsilon^2\right)}. \tag{9}$$

So you can see that determination of the flexural rigidity is equivalent to determining the plate thickness. Now according to equations (7) to (9), the bending moment is just the flexural rigidity of the plate divided by its curvature so

$$M = -D\frac{d^2w}{dx^2} = \frac{D}{R}. \tag{10}$$

We can take the second derivative of equation (9) and substitute it into equation (3) to eliminate the moment. This yields the general expression for the deflection

$$D\frac{d^4w}{dx^4} + P\frac{d^2w}{dx^2} = q(x). \tag{11}$$

Equation (11) is a form of the two-dimensional the flexure equation that includes contributions from vertical and horizontal loads. In practice, we can measure *w* and so we can get an estimate of the effective elastic thickness, *h,* that is contained in *D.*

When we apply a flexural formula to the Earth's lithosphere it is necessary to include the **hydrostatic restoring force** due to the effective replacement of mantle rocks in a vertical column with material of lower density. We first consider the case of oceanic lithosphere. The pressure at the base of deflected lithosphere is

$$\rho_w g(h_w + w) + \rho_m g h \qquad (12)$$

where ρ_w is the density of water and ρ_m is the density of the underlying mantle. The pressure at depth $h_w + h + w$ beneath an undeflected part of the plate is

$$\rho_w g h_w + \rho_m g(h + w). \qquad (13)$$

By equating (12) and (13) we see that there is an upward hydrostatic force per unit area equal to

$$\rho_w g h_w + \rho_m g(h + w) - \left[\rho_w g(h_w + w) + \rho_m g h\right] = (\rho_m - \rho_w) g w \qquad (14)$$

and this tries to restore the plate to its original configuration. The force is upward-directed because more dense mantle material is replaced by less dense water. So the net force per unit area acting on the lithospheric plate is

$$q_{total} = q(x) - (\rho_m - \rho_w) g w$$

where *q* is the applied load at the surface of the plate. Equation (11) thus becomes

$$D\frac{d^4 w}{dx^4} + P\frac{d^2 w}{dx^2} + (\rho_m - \rho_w) g w = q(x) \qquad (15)$$

where *x* is the horizontal length scale, *w* is the vertical displacement, *P* is an applied lateral load such as a tectonic plate stress, *q* is an applied vertical load. An analagous analysis yields the flexure equation for continental lithosphere

$$D\frac{d^4 w}{dx^4} + P\frac{d^2 w}{dx^2} + (\rho_m - \rho_c) g w = q(x) \qquad (16)$$

where ρ_c is the density of continental crust. In practice, equation (15) or (16) is solved for appropriate boundary conditions. Geophysical observations are inverted and fit to models of predicted plate response to loads.

The flexure equation has various solutions for different imposed boundary conditions and these are relatively straightforward to obtain. A number of these solutions are presented in Turcotte and Schubert's text *Geodynamics*. Of particular relevance to marine geophysics are solutions for vertical loads such as seamounts, volcanoes, or sediments, or horizontal loads such as due to lithospheric buckling associated with plate forces.

To first order the mechanical thickness of the lithosphere is well-represented by the simple elastic plate model. There is a considerable body of evidence for elastic behavior of the lithosphere:

- most earthquakes are along plate boundaries, not within plates
- plate motion models that assume rigid rotations of spherical shells generally come close to achieving closed circuits.
- fracture zones and transform faults are fairly straight
- magnetic lineations are quasi-linear, indicating minimal intraplate deformation
- the observed response of the lithosphere fits elastic plate models well in many cases.

As an example of lithosphere flexure, consider the simple case of a load due to a line of volcanic islands such as the Hawaiian chain. We may treat the problem as two dimensional and the line load is then simply $-q_o$. Since the applied load vanishes except at $x=0$ we can simply the problem further by assuming $q(x)=0$ and instead incorporate the load via the boundary conditions. From equation (15) we set $q(x)=0$ and $P=0$ (because there is no horizontal loading in this problem), so the flexure equation reduces to

$$D\frac{d^4w}{dx^4}+(\rho_m-\rho_w)gw=0 \tag{17}$$

which has a general solution

$$w=e^{x/\alpha}\left(c_1\cos\frac{x}{\alpha}+c_2\sin\frac{x}{\alpha}\right)+e^{-x/\alpha}\left(c_3\cos\frac{x}{\alpha}+c_4\sin\frac{x}{\alpha}\right) \tag{18}$$

where the constants c_{1-4} are determined from the boundary conditions and

$$\alpha=\left[\frac{4D}{(\rho_m-\rho_w)}\right]^{1/4} \tag{19}$$

is called the **flexural parameter**. Since the problem is symmetric in $\pm x$ we may limit analysis to $x\geq 0$. Appropriate boundary conditions are $w\to 0$ as $x\to\infty$ and $dw/dx=0$ at $x=0$. Thus $c_1=c_2=0$ (for the solution to be bounded) and $c_3=c_4$ (for symmetry). The general solution (equation 18) reduces to

$$w=c_3e^{-x/\alpha}\left(\cos\frac{x}{\alpha}+\sin\frac{x}{\alpha}\right) \qquad x\geq 0 \qquad (20).$$

The final constant c_3 depends on the magnitude of the applied load q_o. By integrating the flexure equation and realizing $d^3w/dx^3\propto -q_o/D$, it can be shown that

$$c_3=\frac{q_o\alpha^3}{8D} \tag{21}$$

and so

$$w = \frac{q_o \alpha^3}{8D} e^{-x/\alpha}\left(\cos\frac{x}{\alpha} + \sin\frac{x}{\alpha}\right). \tag{22}$$

The maximum amplitude of the deflection occurs at the loaded point at $x=0$ and

$$w_o = \frac{q_o \alpha^3}{8D}$$

and so

$$w = w_o e^{-x/\alpha}\left(\cos\frac{x}{\alpha} + \sin\frac{x}{\alpha}\right). \tag{23}$$

The half-width of the depression is

$$\frac{x_o}{\alpha} = tan^{-1}(-1) = \frac{3\pi}{4}. \tag{24}$$

The distance of the line load to the maximum amplitude of the forebulge x_b is found from where the slope of the profile vanishes. We differentiate equation (23) to find

$$\frac{dw}{dx} = -\frac{2w_o}{\alpha} e^{-x/\alpha} \sin\frac{x}{\alpha} = 0 \tag{25}$$

and we see that

$$\frac{x_b}{\alpha} = sin^{-1}(0) = \pi.$$

Gravity anomalies are an important key to understanding the mechanical behavior of oceanic and continental lithosphere. Flexure analysis that utilizes gravity and topography observations as model constraints is a commonly used technique to determine the mechanical response of the lithosphere to applied loads. Critical in this analysis is the fact that the wavelength of flexure is indicative of the strength of the lithosphere. A strong lithosphere produces a long wavelength of deformation, while a weak lithosphere results in a shorter wavelength but larger amplitude of deformation. The mechanical thickness of oceanic lithosphere varies with age but is on average about 50 km.

The continents, especially in areas of thick crust such as zones of mountain-building, have very low strength. Continental crustal material is sufficiently weak that it cannot support significant loads. Even though mountain belts often have high elevations the usually have small or absent free-air gravity anomalies because the crust beneath the mountain thickens to maintain a constant pressure at some depth. This represents a state called **isostasy** that represents a minimum stress configuration. In areas where topography is isostatic the topography is not supported by the strength of the plate but rather subsurface density differences. The case of isostasy represents the endmember case of an effective elastic lithosphere thickness of $h=0$.

Rheological Lithosphere

A variant of the mechanical lithosphere is the **rheological lithosphere**. This concept utilizes the same philosophy of deriving plate rigidity as for the elastic lithosphere, but invokes more complex representations of mechanical structure that enable analysis of important factors such as temperature dependence. **Rheology** derives from Greek and is the study of flow: it is the branch of physics that deals with the deformation and flow of materials. Since by definition the lithosphere represents the region of the crust and mantle that can support significant stress, the more "fluid-like" rocks become with increasing depth and temperature the less shear stress they can support. Results from rock mechanics experiments and near-surface *in situ* stress measurements imply that rock strength varies with depth in response to changes in ambient temperature, confining pressure and composition.

Frictional Rock Strength

At shallow depth rocks behave in a brittle manner and strength depends mostly on confining pressure, increasing linearly with depth. A general expression for the limiting shear stress τ for slip on randomly oriented fractures in rock may be written

$$\tau = \tau_o + \mu\sigma_n \tag{26}$$

where τ_o is cohesive strength, μ is the coefficient of friction, and σ_n is the effective normal stress. For "real rocks", experiments have mapped out an empirical relationship for the frictional strength of rocks. This is referred to as **Byerlee's Law**, which can be written

$$\begin{aligned} &\sigma_1 \cong 5\sigma_3 && \sigma_3 < 110 \text{ MPa} \\ &\sigma_1 \cong 3.1\sigma_3 + 210 && \sigma_3 > 110 \text{ MPa} \end{aligned} \tag{27}$$

where σ_1 and σ_3 are minimum and maximum principal stresses. In (27) yielding occurs by frictional sliding on fracture surfaces. Byerlee's Law is very general in that it is independent of rock type and grain size.

Ductile Rock Strength

At greater depth temperature effects begin to dominate and rocks will deform by **viscous flow** at lower stresses than they would fracture in a brittle manner. In the viscous creep regime rock strength decreases with increasing temperature according to an Ahrrenius (exponential) relationship.

$$\dot{\varepsilon} = A\sigma^n \exp[-Q/RT] \tag{28}$$

where $\dot{\varepsilon}$ is strain rate, σ is stress, T is temperature, Q is activation energy, R is the gas constant, and A and n are experimentally-determined constants.

Yield Strength Envelope

A particularly useful representation of the rheological lithosphere that incorporates these processes is the **yield stress envelope (YSE)**. The YSE defines the depth-dependence of the maximum supportable stress difference in the lithosphere as a function of depth. The integral of the YSE is indicative of the total strength of the lithosphere and represents the force per unit length that opposes lithospheric failure from imposed horizontal forces.

According to the YSE concept, even small levels of applied force are expected to produce yielding at the top and bottom of the lithosphere, by faulting and ductile creep, respectively. The intervening **strong core** region can maintain large stresses elastically without yielding on geological time scales and correlates in a very general way with the effective elastic thickness. As with the elastic formulation, the core region controls the flexural wavelength of deformation that develops in response to applied loads.

Understanding and interpreting length scales of deformation in the brittle regime is more complicated. Often deformation is controlled by the distribution of existing and pre-existing faults. However, progress in the physics of macroscale localization show that in the absence of a pre-existing tectonic fabric, averaged rheological properties of the brittle layer as well as strain rate and total strain all contribute to the length scales of large-scale faults.

The yield strength envelope for the continents is more complicated for that of the oceans. Continental crust is thicker and more quartz-rich than oceanic crust. Quartz is much weaker than the dominant minerals in oceanic crust. As a consequence the brittle-ductile transition is often reached in the crust and the lower crust undergoes ductile flow. The upper mantle brings the return to brittle fracture, before ductile flow again occurs at depth. This strong upper crust - weak lower crust - strong upper mantle configuration is sometimes referred to as the **jelly sandwich** configuration.

It is worth emphasizing the particular importance of studying the **oceanic lithosphere** in the context of global geodynamics, especially given the fact that in most cases the continents are more readily accessible for study. The previous discussion demonstrates that continents are often more complex than the ocean floor due to tectonic reworking, erosion, and significant compositional heterogeneity, due in part (though not totally) to the continents' greater age. There are certainly complex processes occurring on the ocean floor but they are often (though not always) easier to unravel than on the continents. The clearest surficial exposition of global geodynamics is preserved on the seafloor. But even studying the oceans in this regard has its challenges, as exemplified from analysis of oceanic lithosphere thermal structure.

Thermal Structure of Oceanic Lithosphere

Oceanic lithosphere forms as intruded basaltic magma cools and spreads away from midocean ridges. As expected, heat flow is highest at the ridges and decreases with distance from the ridge. The depth of the seafloor also decreases away from the ridge. This is because as the lithosphere cools it becomes less buoyant and seeks a lower elevation to maintain isostasy. The systematic variation of seafloor depth and heat flow with age is the primary constraint on models of the thermal evolution of oceanic lithosphere.

The cooling of oceanic lithosphere with age away from a spreading ridge requires solution of the heat flow equation:

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2} \tag{29}$$

where T is temperature, t is time, x is distance and $\kappa = k / \rho C_p$ is thermal diffusivity where k is thermal conductivity, ρ is density and C_p is specific heat. Heat flow is often referred to in **heat flow units (hfu)** where 1 hfu = 10^{-6} cal cm^{-2} s^{-1} = 41.84 mWm^{-2}. The mean heat generation H per unit mass M is

$$H = \frac{Q}{M} \tag{30}$$

where Q is the total heat flow. The mean measured heat flow for all of the continents is 56.5 mWm^{-2} (1.35 hfu), with African having the lowest mean value (49.8 mWm^{-2}; 1.19 hfu) and Australia the highest mean (63.7 mWm^{-2}; 1.52 hfu). Regions of high heat flow are often associated with active surface volcanism or volcanic intrusion.

The mean measured heat flow for all ocean basins is 78.2 mWm^{-2} (1.87 hfu) with the North Pacific being the highest (95.4 mWm^{-2}; 2.28 hfu) and the South Atlantic being the lowest (59.0 mWm^{-2}; 1.41 hfu). Because of partitioning during melting, the concentration of heat-producing elements in the oceanic crust is about an order of magnitude less than it is in the continental crust. The contribution of heat production in the oceanic crust to the surface heat flow is only ~2%.

The most interesting feature of heat flow measurements in the oceans is the dependence of heat flow on the age of seafloor. This is due to the gradual cooling of oceanic lithosphere as it moves away from the mid-ocean ridge.

QUESTION: Can cooling alone explain the way that mean depth and heat flow vary with distance from the mid-ocean ridge? We may note two general relationships that have been observed from geophysical observations:

Heat flow relationship: $q \propto \dfrac{dT}{dz}$

where q is heat flow and dT/dz is thermal gradient. The above heat flow relationship comes from Fourier's Law of Heat Conduction, which in one dimension can be written

$$q = -k\frac{dT}{dz}.$$

Bathymetry relationship: $d \propto \displaystyle\int_0^h T dz$

where d is depth, T is temperature, h is the plate thickness, and z is the depth coordinate.

There are two models that have been forwarded to explain the cooling of oceanic lithosphere away from a spreading ridge:

(1) Halfspace model - in this case the lithosphere behaves like the cold upper boundary of a cooling halfspace such that $d \propto (age)^{1/2}$ and $q \propto (age)^{-1/2}$. This requires solution of equation (29) to give

$$T(t,z) = T_m\left[erf\left(z/2\sqrt{\kappa t}\right)\right] \tag{31}$$

where T_m is the mantle temperature, and erf is the error function

$$erf(\eta) \equiv \frac{2}{\sqrt{\pi}}\int_0^\eta e^{-\eta'^2} d\eta'. \tag{32}$$

Values of (9) are tabulated in many math tables. The corresponding heat flow q is

$$q(t) = \frac{kT_m}{\sqrt{\pi\kappa t}} \tag{33}$$

and the ocean depth varies as distance from the spreading ridge as

$$d(t) = d_r + 2\sqrt{\frac{\kappa t}{\pi}}\frac{\alpha\rho_m T_m}{(\rho_m - \rho_w)} \tag{34}$$

where d_r is the depth of the mid-ocean ridge, α is the coefficient of thermal expansion and the subscripts m and w refer to mantle and water, respectively.

(2) Plate model - in this case the lithosphere behaves like a cooling plate with an isothermal lower boundary until it reaches an age at which the effect of the lower boundary causes depth and heat flow to flatten and vary more slowly with age. Plate cooling can be described by

$$T(x,z) = T_m\left[\frac{z}{a} + \sum_{n=1}^{\infty} c_n \exp\left(\frac{\beta_n x}{a}\right)\sin\left(\frac{n\pi z}{a}\right)\right] \tag{35}$$

where a is the asymptotic thermal plate thickness, T_m is the basal temperature and $c_n = 2/(n\pi)$, $\beta_n = \left(R^2 + n^2\pi^2\right)^{1/2} - R$, $R = va/(2\kappa)$ is the Peclet number that relates the advective and conductive heat transfer, and v is the half-spreading rate. The surface heat flow is

$$q(x) = q_s + \left[1 + 2\sum_{n=1}^{\infty}\exp\left(-\frac{\beta_n x}{a}\right)\right] \tag{36}$$

wghere $q_s = kT_m / a$ is the asymptotic heat flow for old lithosphere, in which the thermal gradient is linear. The ocean depth is

$$d(x) = d_r + d_s\left[1 - \frac{8}{\pi^2}\sum_{j=1}^{\infty} j^{-2}\exp\left(-\frac{\beta_n x}{a}\right)\right] \tag{37}$$

where j takes only odd values and

$$d_s = \frac{\alpha\rho_m T_m a}{2(\rho_m - \rho_w)} \tag{38}$$

is the asymptotic subsidence of old oceanic lithosphere.

Physically, the plate model is usually interpreted as having a source of heat at depth whose effect “kicks in” at lithosphere of a certain age. Possibilities for this additional heat source include:

- radiogenic heating
- small-scale convection
- shear heating
- mantle plumes.

It is observed that at 63 Ma the depth curve for oceanic lithosphere does flatten. The plate model is the simplest model (in terms of numbers of parameters) that can describe the general variation of depth and heat flow. One might think that this would settle the question. But there are significant variants and the thermal structure is not well understood. Even among proponents of the plate model there is disagreement as to what is the thickness of the plate -- *i.e.* the base level behavior has not been well established.

Part of the problem is that heat flow measurements are difficult to interpret due to complications such as **hydrothermal circulation**. There is also disagreement as to which heat values are "anomalous" *i.e.* excludable from the trend due to effects other than the cooling of age, such as due to hotspots. As for interpretation of depth data, there is question how much depth anomalies are attributable to thermal vs. compositional effects.

There is a possibility that systematic analysis of the oceanic geoid might provide additional information that could help with the non-uniqueness problem. The oceanic geoid reflects the depth—weighted integral of density anomalies, and thus the weighted integral of the geothermal gradient. Note that the slope of the geoid is proportional to

$$\frac{\partial}{\partial t}\int zT(z,t)dz$$

and so it reflects

$$\alpha T_m \exp(-t/a^2)$$

where α is the coefficient of thermal expansion, T_m is mantle temperature and a is plate thickness. But geoid observations for various places on the Earth have not been processed in a uniform manner so the task is daunting.

Hydrothermal Circulation

Many anomalously low values of heat flow are found in oceanic crust in the age range of 3-4 Ma. Beyond 5 Ma the values settle somewhat; there is less scatter and the values fall relatively close to theoretical curves. Oceanic crust forms by intrusion of basaltic flows and dikes. When magma comes into contact with seawater it quenches, produces large thermal stresses that lead to contraction, and cracks are created which brings more warm material in contact with seawater. The penetration of this effect is felt to depths as high as 6-7 km. It is limited by overburden pressure due to the weight of overlying material which must be overcome for fracture to occur.

When sediments, which are fairly impermeable, are laid down, they blanket the oceanic crust. Hydrothermal circulation still occurs, but no heat is loss by advection, which is the physical transport of warm material. In such areas the mean heat flow that is measured is a reasonable estimate of the heat flow at depth.