Spacing of Faults at the Scale of the Lithosphere and Localization Instability 1: Theory

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Abstract. Large-scale tectonic structures such as orogens and rifts commonly display regularly–spaced faults and/or localized shear zones. Numerical models reproduce this phenomenon. To understand how fault sets organize with a characteristic spacing, we present a semi–analytical instability analysis of an idealized lithosphere composed of a brittle layer over a ductile half–space undergoing horizontal shortening or extension. The rheology of the layer is characterized by an effective stress exponent, \( n_e \). The layer is pseudo–plastic if \( 1/n_e = 0 \) and forms localized structures if \( 1/n_e < 0 \). The tendency for localization is stronger for more negative \( 1/n_e \). Two instabilities grow simultaneously in this model: the buckling/necking instability that produces broad undulations of the brittle layer as a whole, and the localization instability that produces a spatially periodic pattern of faulting. The latter appears only if the material in the brittle layer weakens in response to a local perturbation of strain rate, as indicated by \( 1/n_e < 0 \). Fault spacing scales with the thickness of the brittle layer and depends on the efficiency of localization. The more efficient localization is, the more widely spaced are the faults. The fault spacing is related to the wavelength at which different deformation modes within the layer enter a resonance that exists only if \( 1/n_e < 0 \). Depth–dependent viscosity and the model density offset the instability wavelengths by an amount that we determine empirically. The wavenumber of the localization instability, is \( k_n^L = \pi (j + a_L)/(-1/n_e)^{-1/2}/H \), with \( H \) the thickness of the brittle layer, \( j \) an integer, and \( 1/4 < a_L < 1/2 \) if the strength of the layer increases with depth and the strength of the substrate decreases with depth.

1. Introduction

Much of our theoretical understanding of tectonics stems from continuum mechanics [Turcotte and Schubert, 2002]. For instance, certain large–scale patterns of deformation resemble a mode of folding of the strong layers of the lithosphere called buckling in compression and necking in extension [Biot, 1961; Fletcher and Hallet, 1983; Ricard and Froidevaux, 1986; Zuber et al., 1986; Zuber, 1987]. The buckling or necking theory predicts a preferred wavelength of deformation that is controlled by the mechanical structure of the lithosphere. Hence, recognizing buckling in the geological records helps to constrain the structure of the lithosphere at the time when the structure formed.

Faults and shear zones constitute another primary indicator of the structure of orogens or rifts that can be related to continuum mechanics models [Beaumont and Quinlan, 1994]. Faults often present a preferred spacing or characteristic scale [Weissel et al., 1980; Zuber et al., 1986; Davies, 1990; Watters, 1991; Bourne et al., 1998]. The buckling/necking theory might then be used to model fault spacing [Watters, 1991; Brown and Grimm, 1997], but faults actually represent a localized style of deformation that is not accessible using the continuum theories implied in the buckling/necking analysis; deformation occurs mostly—if not entirely—within a narrow band. In this study, we modify the buckling/necking theory to consider explicitly the dynamics of localization, and to explore the link between the structure of the lithosphere and patterns of localized shear zones or faults.

Buckling and necking produce broad undulations of the lithosphere in which deformation is distributed. The pseudo–plastic rheology used to model the brittle levels of the lithosphere [Fletcher and Hallet, 1983] assumes distributed failure or faulting. However, faulting has a tendency to localize, to abandon a distributed style of faulting to concentrate deformation on a few isolated major faults [Sornette and Vanneste, 1996; Gerbault et al., 1998]. As stress heterogeneities induced by buckling favor faulting in the hinge of large–scale folds [Lambeck, 1983; Martinod and Davy, 1994; Gerbault et al., 1999], localized fault patterns can be controlled by buckling if they develop after the folds have reached sufficient amplitude. However, faulting may occur from the onset of deformation and hence, may develop without the influence of finite–amplitude buckling. Indeed, some tectonic provinces display faults with a characteristic spacing unrelated to the buckling wavelength. To cite only examples in compressive environments, faults in the Central Indian Basin [Bull, 1990; Van Orman et al., 1995], in Central Asia [Nikishin et al., 1993], or in Venetian fold belts [Zuber and Aist, 1990] are more closely spaced than the wavelength of folds in the same region. In these regions, faulting and buckling appear as superposed deformation styles, each with a characteristic length scale.
Buckling and necking were first studied in Earth sciences as a mechanism to form folds or boudins in outcrop–scale layered sequences [Johnson and Fletcher, 1994]. While originally derived using a thin plate approximations of viscous and/or elastic materials [Ramberg, 1961; Biot, 1961], the buckling/necking theory was later developed with a thick plate formulation [Fletcher, 1974; Smith, 1975] and was applied to non–Newtonian materials [Fletcher, 1974; Smith, 1977]. For a non–linear rheology, the stress supported by the fluid, $\sigma$, is related to the second invariant of strain rate, $\dot{e}_{11}$, by $\dot{e}_{11} \propto \sigma^{n_e}$, with $n_e$ the effective stress exponent, a measure of the non–linearity of the rock rheology [Smith, 1977; Montesi and Zuber, 2002]. Non–Newtonian creep with $1 < n_e < 5$ is the rheology that describes rocks at sufficiently high temperature to behave in a ductile manner.

At low temperature, rocks behave instead in a brittle manner. The stress that they can support is limited by a yield strength, at which failure, faulting, and plastic flow occur. Yielding can be included in thin–plate analyses of folding by limiting the bending stresses to the yield strength and reducing the apparent flexural rigidity of an elastic or viscous plate accordingly [Chapple, 1969; McAdoo and Sandwell, 1985; Wallace and Melosh, 1994]. The thick–plate formulation of the buckling theory is particularly well adapted to an alternative treatment of failure, in which the yield material is approximated as a highly non–Newtonian fluid in the limit $n_e \to +\infty$ [Chapple, 1969, 1978; Smith, 1979]. Most lithospheric–scale applications of buckling use that approximation [Fletcher and Hallet, 1983; Zuber et al., 1986; Zuber, 1987]. More accurate treatments of yielding have been included in buckling/necking theory. Leroy and Triantafyllidis [1996, 2000] and Triantafyllidis and Leroy [1997] studied the necking behavior of a hardening elastic–plastic medium at yield using the strain rate–stress rate relations of the deformation theory of plasticity. Localized faulting is predicted only for tectonic stresses is excess of the critical value for necking and is never predicted if the flow theory of plasticity is used [Triantafyllidis and Leroy, 1997]. Therefore this model cannot explain regions that show regularly–spaced localized faulting. Davies [1990] modeled the buckling of a rigid–plastic layer with associated flow law surrounded by a rigid basement and a viscous half–space. Faults were forced by a local cusp in the model interface but an initially distributed perturbation remains distributed. Neumann and Zuber [1995] found that a localized perturbation triggers macroscale shear bands in a power–law medium with $n_e \to +\infty$ as well. Fletcher [1998], who also included the effects of pore fluids and pressure solution on a non–Newtonian porous fluid with $n_e \to +\infty$, showed that the shear bands produced by a localized forcing are ephemeral. He speculated that the bands would be stabilized and therefore may correspond to faults if strain–weakening were included [Fletcher, 1998].

All previous treatments of yielding in buckling theory fail to produce localized deformation; faulting remains distributed throughout the material. Hence, any fault pattern observed in nature is expected to develop late in the folding history, with a spacing that is controlled by the buckling or necking wavelength. However, this is contrary to many geological observations [Weissel et al., 1980; Nikishin et al., 1993; Krishna et al., 2001]. In order to understand how fault spacing may differ from the buckling/necking wavelength, we study the buckling behavior of simplified lithosphere models with a rheology that weakens with deformation upon yielding. The weakening behavior is characterized by a negative effective stress exponent. Such an effective rheology brings a tendency to localize deformation and faulting [Montesi and Zuber, 2002].

The effective stress exponent, $n_e$, indicates how a material responds to a perturbation in the deformation field [Smith, 1977]. In this study, deformation is quantified by the second strain rate invariant, $\dot{e}_{11}$. When $n_e < 0$, increasing $\dot{e}_{11}$ decreases the material strength, which ensures localization [Montesi and Zuber, 2002].

The (algebraic) value of $n_e$ is determined from the physical process that produces localization. It incorporates not only the direct effect of perturbing the strain rate, but also the possible feedback of internal variables that control the rheology. For instance, a frictional material such as the pervasively faulted brittle lithosphere requires a higher stress to deform more rapidly, which would result in positive $n_e$. However, a higher sliding velocity on faults changes also the physical state of a granular gouge inside the fault and results in apparent weakening and negative $n_e$. [Dieterich, 1979; Scholz, 1990]. In Montesi and Zuber [2002], we derived the conditions for which this feedback mechanism and others produce localization, and we gave values for the corresponding effective stress exponents. For frictional sliding, $-300 < n_e < -50$.

Neurath and Smith [1982] showed that strain weakening reduces the value of $1/n_e$ in a non–Newtonian fluid, possibly resulting in $n_e < 0$. However, the materials that they studied never had negative $n_e$. While they recognized that $n_e < 0$ would lead to “catastrophic failure”, or localization, modulated by the interaction with the surroundings of the layer with negative exponent, Neurath and Smith [1982] did not present an analysis of buckling with $n_e < 0$.

In this paper, we derive the growth spectrum for simple models of the lithosphere, which indicates how fast a given wavelength grows in a particular model [Johnson and Fletcher, 1994]. Buckling and necking occur at wavelengths at which the growth rate is maximum. We identify a new instability when the material localizes and call it localization instability. Both the localization instability and the buckling/necking instability are associated with particular resonances within the localizing layer. We show how the wavelengths of the buckling and localization instabilities relate to resonances within the model, considering several types of depth–dependent strength profiles that approximate the strength profile of a mono–mineralic lithosphere. Most of this study is conducted assuming uniform horizontal shortening, with a generalization to horizontal extension in §6.2. In a companion paper, we show that the localization instability is a possible explanation for fault spacing in the Central Indian Basin, which is incompatible with a buckling instability.

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**Figure 1.** Schematic of lithosphere models. A layer of thickness $H$, viscosity $n_1(z)$, and effective stress exponent $n_1$ lies over a half–space of different mechanical properties ($n_2(z)$, $n_2 = 3$). The model is two–dimensional, incompressible, and subjected to pure shear shortening at the rate $v_{xz}$. Its density is $\rho$. Boundary conditions are no slip at the interface, no stress at the surface. For the instability analysis, the interfaces are perturbed by an infinitesimal sinusoidal topography of wavelength $\lambda$ and amplitude $\xi_1$ and $\xi_2$.  

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2. Instability principle

2.1. Solution strategy

The buckling/necking and localization instabilities develop in mechanically layered models of the lithosphere undergoing horizontal shortening or extension. In this study, we concentrate on the behavior of a single brittle or plastic layer overlying a ductile half–space under horizontal shortening at the rate $\dot{\varepsilon}_{xz}$ (Figure 1). Horizontal extension is discussed in §6.2. Each material is incompressible. As the deformation field is constrained to be two–dimensional, it is completely determined by the stream function $\varphi(x, z)$.

The deformation field in each layer is decomposed into a primary field and a secondary field of much smaller amplitude. The primary field is the flow solution when the interfaces in the model are perfectly flat and horizontal. It is invariant in $x$, with $\varepsilon_{zz} = -\varepsilon_{xz}$, and $\varepsilon_{xx} = 0$. However, any relief on the interfaces perturbs the system and drives a secondary flow in the model. In turn, the secondary flow deforms the interface.

This analysis determines the rate at which interface perturbations grow in a given model. The flow equations for the secondary deformation field and the boundary conditions are linearized, assuming that the interface perturbations have a much smaller amplitude than the layer thickness [Johnson and Fletcher, 1994; Montési, 2002]. Thus, the Fourier components of an infinitesimal generic perturbation are decoupled from one another. Hence, we consider interface perturbations of the form

$$\xi_i = \xi_i^0 \exp(ikx),$$

with $\xi_i^0$ the amplitude of the perturbation, $k$ its wavenumber, $i$ an index identifying an interface of the model, and $i = (-1)^{1/2}$. There are several modes of deformation in each layer, each characterized by a stream function $\varphi_j$

$$\varphi_j = \varphi_j^0 \phi_j(z) \exp(ikx),$$

where $\phi_j$ is the depth kernel, $\varphi_j^0$ is the amplitude of $\varphi_j$, and $j$ is an index identifying each deformation mode. The depth kernel is determined from the equations of Newtonian equilibrium and the index identifying each deformation mode. The depth kernel is an analytic function of depth in each layer. The associated eigenvector describes the fastest rate of perturbation.

The rate at which the interface perturbations grow has a kinematic contribution from the pure shear thickening of the model (primary flow field) and a dynamic contribution from the secondary flow field [Smith, 1975]. We obtain:

$$\frac{d\xi_i}{dt} = (-\dot{\varepsilon}_{xz} \delta_{ij} + Q_{ij}) \xi_j,$$

with $\delta_{ij}$ the Kronecker operator and $Q_{ij}$ the growth matrix, which depends on $k$ and the strength and density structure of the model [Montési, 2002]. The growth rate $Q$ is the eigenvalue of $Q$ that has the largest real part. The associated eigenvector describes the fastest growing deformation mode of the model as a whole [Smith, 1975; Zuber et al., 1986]. For instance, it determines whether a given layer deforms by buckling (upper and lower interfaces in phase) or necking (upper and lower interfaces out of phase). The growth spectrum is defined as the function $Q(k)$.

2.2. Effective rheology and effective stress exponent

The strength of each layer of the lithosphere model is an analytical function of depth. As we address the organization of strain rate within this model, we define the apparent viscosity $\eta$ by

$$\sigma_{ij} = -p\delta_{ij} + \eta \dot{\varepsilon}_{ij},$$

where $\sigma$ is the stress supported by the material, $p$ the pressure, and $\dot{\varepsilon}$ the strain rate. In general, $\eta$ depends on the second invariant of the strain rate, $\dot{\varepsilon}_{II}$ and on the depth $z$. In the brittle field, the material strength and therefore the viscosity may also depend on the strain undergone by the material. In this study, we chose to ignore that additional complication, which would also require elasticity to be included in the model and make the analysis time–dependent [Neurath and Smith, 1982; Schmalholz and Podladchikov, 1999, 2001].

In the perturbation analysis, the primary field, which represents the state of uniform shortening, is denoted by over–bars. It obeys

$$\dot{\sigma}_{ij} = -p\delta_{ij} + \eta \dot{\varepsilon}_{ij},$$

with $\eta$ the material viscosity at the externally imposed strain rate $\dot{\varepsilon}_{xz} = |\dot{\varepsilon}_{xz}|$. It is an analytic function of depth in each layer. The secondary field, denoted with a tilde, which represents the perturbing flow obeys the apparent rheology:

$$\ddot{\sigma}_{xx} = -\ddot{p} + \frac{\eta}{n_e} \dot{\varepsilon}_{xx},$$

$$\ddot{\sigma}_{zz} = -\ddot{p} + \frac{\eta}{n_e} \dot{\varepsilon}_{zz},$$

$$\ddot{\sigma}_{xz} = \eta \ddot{\varepsilon}_{xz},$$

with $n_e$ the effective stress exponent,

$$\frac{1}{n_e} = 1 + \frac{\ddot{\varepsilon}_{zz}}{\eta} \frac{\partial \eta}{\partial \varepsilon_{zz}} + \frac{\ddot{\varepsilon}_{xz}}{\eta} \frac{\partial \eta}{\partial \varepsilon_{xz}}.$$
describing damage, or state of a fault gouge. This variable may require a finite time to respond to a local variation of strain rate. This results in an immediate strengthening response of the system, followed by weakening in the long-time limit. In this study, we ignore the transient response, arguing that perturbations may be held for long enough that steady-state is reached, and that the perturbation amplitude is so small that the strengthening “barrier” is easily overcome. However, this assumption should be relaxed in future work.

Most previous studies of lithospheric–scale instabilities treated the brittle upper crust and mantle as pseudo–plastic with $1/n_e \to 0$°. These instabilities produce buckling in compression, and necking in extension [Fletcher and Hallet, 1983; Ricard and Froideveau, 1986; Zuber, 1987]. In this study, we introduce the solutions for $1/n_e < 0$, which produce regularly–spaced shear zones, through a process that we call localization instability. The more negative $1/n_e$ is, the stronger localization is. Intuitively, a more efficient localized shear zone can accommodate the deformation from a wider non–localized region of the lithosphere. Hence, the spacing of localized shear zones should increase when $1/n_e$ is more negative.

3. Depth kernel

3.1. Fundamental equation

The first step in solving the instability problem defined above is to determine the expression of the depth kernel, $\phi(z)$, which gives the depth–dependence of the stream function (Eq. 2) and therefore of the deformation field for each mode of deformation. Whereas the strength and the effective viscosity the lithosphere depend on depth, its effective stress exponent does not necessarily do so, as it measures the rate at which a rock weakens, scaled by its strength. In fact, $n_e$ does not depend on depth for most processes of localization [Montési and Zuber, 2002]. Therefore, we make the assumption that $n_e$ is independent of depth, $z$, within each layer.

We write the equations of Newtonian equilibrium for the secondary flow, using its apparent rheology (Eq. 6), and expressing the strain rate as a function of the stream function. These equations are then combined to eliminate the pressure term, and simplified as their dependence in $x$ is $\exp(ikx)$. We obtain [Montési, 2002]

$$\frac{d^4\phi}{dz^4} + 2r^2 \frac{d^3\phi}{dz^3} - (Ak^2 - s) \frac{d^2\phi}{dz^2} - Ak^2 e^s \frac{d\phi}{dz} + (k^4 + k^2s) \phi = 0,$$

(8)

where we used the notation

$$r(z) = \frac{dn_e}{dz},$$

(9)

$$s(z) = \frac{d^2n_e}{dz^2}$$

(10)

$$A = \frac{4}{n_e} - 2.$$  

(11)

Eq. 7 admits four solutions for a given strength profile $\eta(z)$, effective stress exponent $n_e$, and wavenumber $k$. Hence, there are four superposed deformation modes in each layer, for a given wavelength, each with its own amplitude that is determined from matching boundary conditions at each interface [Montési, 2002].

3.2. Depth kernel for exponential and constant viscosity profiles

The depth kernel can be determined analytically if the viscosity varies exponentially with depth. Then, the function $r$ does not depend on $z$, $s = r^2$, and

$$\eta = \eta_e \exp(rz),$$

(12)

with $\eta_e$ a constant. Constant viscosity layers are included as the special case $r = 0$.

For an exponential viscosity profile, the depth kernel takes the form

$$\phi = \phi_0 \exp i\alpha k z,$$

(13)

with

$$\alpha = \frac{R}{2} \pm \left[1 - \frac{2}{n_e} - \frac{R^2}{4} \pm \left(\frac{4}{n_e} - \frac{R^2}{4}\right)^{1/2}\right]^{1/2}$$

(14)

where $R = r$ [Fletcher and Hallet, 1983].

There are four values of the parameter $\alpha$, each corresponding to a given deformation mode with spatial dependence: $\exp[ik(x + az)]$. Therefore, the amplitude of the stream function varies over a depth scale $1/\Im\alpha(e)$ and is correlated along lines with slope $-\Re\alpha$. Hence, $\alpha$ is referred to as the mode slope.

For constant viscosity layers ($r = 0$), Eq. 14 becomes

$$\alpha = \begin{cases} 
\frac{\pm i}{1} \sqrt{1 + 2 (1 \pm \sqrt{1 - n_e}) / \eta_e}, & \text{if } 1 < 1/n_e, \\
\frac{\pm 1 - 1/n_e \pm i}{1/n_e}, & \text{if } 0 < 1/n_e < 1, \\
\frac{\pm 1 - 2 (1 \pm \sqrt{1 - n_e}) / \eta_e}, & \text{if } 1/n_e < 0.
\end{cases}$$

(15)

By convention, we define $\alpha_1, \alpha_2, \alpha_3, \text{and } \alpha_4$, by selecting in Eq. 14 or 15 the sign combinations $(+, +), (++, -), (-, +), \text{and } (-, -)$. Note that $\alpha$ is real when $n_e$ is negative, i.e., when the material localizes. In that case, the stream function is correlated along four different slopes, but its amplitude is constant with depth: interface perturbations generate four wavelike deformation fields in each layer, none of which decays or grows with depth. This makes it impossible to solve for the behavior of a half–space with negative $n_e$, which requires the deformation field to vanish at infinity. Hence, the simplest solvable model that includes negative $n_e$ is made of a layer of finite thickness over a half–space with positive $n_e$. Although our formulation can in principle handle any number of layers, only this type of model is considered in this paper.

The depth kernel for an exponential viscosity profile with $r \neq 0$ is discussed in Montési [2002]. The domain where all $\alpha$ are real is pushed to more negative $1/n_e$ when $R$ increases, and vanishes altogether at $R = 2\sqrt{2} + \sqrt{3}$. This is a limit where the viscosity increases rapidly compared to the perturbation wavelength. However, even in the large wavelength limit, there are more than two values $\Re\alpha$, one of which is zero. We will see that this condition is sufficient for the localization instability to develop.

3.3. General solution

For a non–exponential viscosity profile, Eq. 7 must be solved numerically. We use a Runge–Kutta integration technique [e.g., Hamming, 1973]. By convention, the depth kernel has a value of 1 at the top of each layer. The four superposed modes of deformation are found by setting the initial values of the depth–derivatives of $\phi$ in turn to each solution of $\phi(z)$ for an exponential viscosity profile that approximates the actual viscosity profile at the top of the layer (Eq. 14). We verified that the solutions do not depend on the actual starting scheme chosen. The exceptional cases where the solutions are degenerate are ignored, and do not arise in practice except if $1/n_e = 1$ or $1/n_e = 0$. 

Figure 2. a) Growth spectrum for a layer of uniform viscosity $\eta_1$ overlying a half-space of uniform viscosity $\eta_2 = \eta_1/10$, with $\rho = 0$, $n_2 = 3$. Solid line: $n_1 = -10$; dashed line: $n_1 = 10^6$; dotted line: $n_1 = 100$. b) Viscosity profile. $K_{BN} \approx \pi$ and $K_L \approx 10$ for the values of $n_1$ considered.

Figure 3. Deformation fields corresponding to the growth spectra in Fig. 2, shaded as a function of strain rate. a) $n_1 = 10^6$; b) $n_1 = -10$. The velocity field is computed from a small-amplitude initial perturbation at the surface such that $\xi_1 \propto k^b$, with $b$ a real number. Then, the amplitude of the other interface is set to give the eigenvector of the growth matrix (Eq. 3) with the fastest growth rate. These modes are amplified by the growth rate (limited to an arbitrary value if $n_1 < 0$). The initial spectral shape, the amplification factor, and the limiting growth rate have been chosen to show clearly (a) the buckling deformation mode and (b) both the buckling and the localization modes.

We use the numerical solver only for the case where the viscosity depends linearly with depth. To our knowledge, only Bassi and Bonnin (1988) have considered that case previously. They used a polynomial expansion of the depth kernel and determined a recurrence relation between the polynomial coefficients. Although technically an analytical solution, this scheme is subject to numerical errors and truncation of the expansion. We favored the numerical integration technique as it can handle any viscosity profile such as in the companion paper. The only limitation is that $\eta \neq 0$. 
4. Constant viscosity analysis

4.1. Growth spectra

We first consider models where a layer of thickness $H$, effective stress exponent $n_1$, and viscosity $\eta_1$, independent of depth, overlies a half-space of lower viscosity $\eta_2$ and effective stress exponent $n_2 > 0$. The exact value of $n_2$ matters little. In our reference model, we use $n_2 = 3$ and $\eta_2 = \eta_1/10$ (Figure 1). This is the simplest approximation of the lithosphere strength structure that displays both localization and buckling instabilities. The model is shortened at the rate $\dot{\varepsilon}_2 < 0$. For now, the material density is ignored. The coordinates $x$ and $z$ are normalized by $H$, the wavenumbers are normalized by $H^{-1}$, and the stresses are normalized by $\eta_1 \dot{\varepsilon}_2$.

We present in Figure 2 the growth spectra for various values of the effective stress exponent of the layer: $1/n_1 = 10^{-2}, 10^{-6}$, and $-10^{-1}$. The flow fields for the last two cases are plotted in Fig. 3.

The cases with positive $n_1$ have been solved before [Ricard and Froidevaux, 1986; Zuber, 1987]. The growth spectrum passes through a first maximum at wavenumber $k \approx \pi/2$ that defines the preferred wavelength of the buckling instability of the layer as a whole (Fig. 3a). A growth spectrum characteristic of the buckling instability passes through successive maxima with a wavenumber scale $K_{B/N}$—standing for wavenumber of the buckling/necking instability. If $1/n_1$ is finite, the envelope of the growth spectrum decreases with wavenumber following the decay of the secondary field with depth indicated by $\text{Im}(\alpha) \neq 0$ in Eq. 15 [Ricard and Froidevaux, 1986; Neumann and Zuber, 1995]. Accordingly, the magnitude of the growth rate does not change with $k$ in the pseudo-plastic limit $1/n_1 \to 0$, represented here by $1/n_1 = 10^{-6}$, as $\text{Im}(\alpha) = 0$ in that case.

The growth spectrum for the case $n_1 = -10$ is different from the other two (Fig. 2). It is best described as the superposition of a buckling-type spectrum and a sequence of doublets with infinite growth rate. These doublets indicate the localization instability. The difference between the wavenumbers of consecutive doublets defines the wavenumber scale $K_L$, which is different from $K_{B/N}$. In Fig. 2, $K_{B/N} \sim \pi$ and $K_L \sim 10$. The reconstructed deformation field (Fig. 3b) has the same appearance of two superposed deformation modes, each having a specific length scale: buckling of the layer as a whole, and regularly-spaced localized shear zones.

The infinite growth rate at the localization doublets (Fig. 2) is due to the unstable character of localization: a local perturbation of strain rate weakens the material locally, so the strain rate increases further. The strain rate perturbations trigger a positive feedback that results in an infinite growth rate. The localized shear zones have a large-scale organization given by the wavenumber at which a divergent doublet is present.

4.2. Resonance

The wavenumber scales $K_{B/N}$ and $K_L$ apparent in the growth spectra (Fig. 2) correspond to certain resonances between the superposed deformation modes in the layer of our model. If the wavelength of the interface perturbation is $\lambda$, potential shear zones are nucleated at the top surface ($z = 1$, as lengths are normalized by $H$) at $x = x_0 + j \lambda$, $j$ an integer ($i.e., j \in \mathbb{Z}$). At each location,
four shear zones propagate into the medium, each corresponding to a particular deformation mode, or solution of Eq. 7. A shear zone nucleated at \( x = x_0, z = 1 \), reaches the bottom of the layer \( (z = 0) \) at \( x_1 = x_0 - \text{Re}(\alpha_1) \), where \( \alpha_1 \) is an index between 1 and 4 that indicates which mode slope \( \alpha \), or solution of Eq. 15, is considered. A second shear zone, identical to the first, is generated at \( x = j\lambda \). It reaches the bottom of the layer at \( x_2 = x_0 + j\lambda + \text{Re}(\alpha_2) \), where \( \alpha_2 \) is another index between 1 and 4, not necessarily the same as \( \alpha_1 \). A given wavenumber \( k = 2\pi/\lambda \) is resonant if \( x_1 = x_2 \), or

\[
k_{\alpha_1,\alpha_2}^j = 2\pi j/|\text{Re}(\alpha_1 - \alpha_2)|,
\]

where \( j \) is the order of the resonance—the number of wavelengths spanned between the surface intercept of the incipient shear zones. The resonant wavenumbers are plotted as a function of the effective stress exponent in Fig. 4.

At depth, each potential shear zone attempts to generate a discontinuous deformation field that is incompatible with the response of the substrate. Therefore, the localized shear zones can develop only if an additional degree of freedom is available, which happens when several shear zones interact at the bottom of the layer. Hence, the localized shear zones cannot develop in the model unless the perturbation wavelength is resonant. Indeed, the growth rate is finite at all wavenumbers that are not resonant, indicating that the model is stable and deformation remains distributed (Fig. 2). At resonant wavenumbers, there is a self-consistent network of narrow zones of high strain rate and the growth rate is infinite.

The pattern of resonances compares well to the buckling and localization instabilities. We present in Fig. 5 a map of growth rate in the parameter space of \( 1/n_1 = k \). This representation is akin to a topographic map of the surface intercept of the incipient shear zones. The resonant wavenumbers are plotted as a function of the effective stress exponent in Fig. 4.

The growth rate maxima are located at

\[
k_j^B = (j + 1/2) K_{B/N}, \quad j \in \mathbb{Z}.
\]

The localization instability is seen as a branch of very high growth rate that exists only when \( 1/n_e < 0 \). Its characteristic wavenumber, \( K_L \), follows a different resonance from the buckling instability

\[
K_L = k_{1,2} = \pi (-1/n_1)^{-1/2}.
\]

The wavenumber \( K_L \) is real only if \( 1/n_1 < 0 \), as expected if it arises due to localization. The growth rate maxima for the localization instability are located near

\[
k_j^L = (j) K_L, \quad j \in \mathbb{Z}.
\]

The relevant resonances are shown schematically in Fig. 6.

As was pointed out earlier, the effective stress exponent quantifies the efficiency of the localization process in the layer. Hence, the wavelength of the localization instability should depend on \( n_e \). The deformation imposed within one wavelength of a given fault is localized on that fault. Therefore, if localization is efficient \( (1/n_e \) more negative), a given fault can accommodate the deformation from a wider area than if localization is inefficient \( (1/n_1 \) less negative). In the limit of perfect localization, all the deformation is localized on a single fault. Indeed, the wavelength of the localization instability goes to infinity in the limit of drastic localization

\[
1/n_e \to -\infty \text{ (Fig. 5)}.
\]

In the other case where the localization is marginal, \( 1/n_e \to 0^- \), the instability wavelength goes to 0; many finely-spaced faults are predicted.

As the localization instability is characterized by a doublet of divergent growth rate, centered on the wavenumber given by Eq. 20, there are two preferred wavelengths of the localization instability for a given \( k \). The separation of the two branches of a given doublet is not predicted by the resonance analysis. We note that the doublets close when several resonances are superposed (compare Fig. 4 and 5), which might indicate the importance of the derivatives of the stream function. Whereas the resonance analysis uses only the stream function, the boundary conditions include velocity and stresses, which depend on the derivatives of the stream function.

Two modes of deformation with different mode slopes have different values of stress and velocity at a given depth, even if their stream functions at that depth are identical. This might suffice to offset

\[
K_{B/N} = k_{1,A} = \pi (1 - 1/n_1)^{-1/2}.
\]

The growth rate maxima are located at

\[
k_j^B = (j + 1/2) K_{B/N}, \quad j \in \mathbb{Z}.
\]

The localization instability is seen as a branch of very high growth rate that exists only when \( 1/n_e < 0 \). Its characteristic wavenumber, \( K_L \), follows a different resonance from the buckling instability

\[
K_L = k_{1,2} = \pi (-1/n_1)^{-1/2}.
\]

The wavenumber \( K_L \) is real only if \( 1/n_1 < 0 \), as expected if it arises due to localization. The growth rate maxima for the localization instability are located near

\[
k_j^L = (j) K_L, \quad j \in \mathbb{Z}.
\]

The relevant resonances are shown schematically in Fig. 6.

As was pointed out earlier, the effective stress exponent quantifies the efficiency of the localization process in the layer. Hence, the wavelength of the localization instability should depend on \( n_e \). The deformation imposed within one wavelength of a given fault is localized on that fault. Therefore, if localization is efficient \( (1/n_e \) more negative), a given fault can accommodate the deformation from a wider area than if localization is inefficient \( (1/n_1 \) less negative). In the limit of perfect localization, all the deformation is localized on a single fault. Indeed, the wavelength of the localization instability goes to infinity in the limit of drastic localization

\[
1/n_e \to -\infty \text{ (Fig. 5)}.
\]

In the other case where the localization is marginal, \( 1/n_e \to 0^- \), the instability wavelength goes to 0; many finely-spaced faults are predicted.

As the localization instability is characterized by a doublet of divergent growth rate, centered on the wavenumber given by Eq. 20, there are two preferred wavelengths of the localization instability for a given \( j \). The separation of the two branches of a given doublet is not predicted by the resonance analysis. We note that the doublets close when several resonances are superposed (compare Fig. 4 and 5), which might indicate the importance of the derivatives of the stream function. Whereas the resonance analysis uses only the stream function, the boundary conditions include velocity and stresses, which depend on the derivatives of the stream function.

Two modes of deformation with different mode slopes have different values of stress and velocity at a given depth, even if their stream functions at that depth are identical. This might suffice to offset
slightly the actual resonant wavelengths. The opening of the localization doublets is usually small enough to be ignored. In addition, the doublet structure may break down once the non-linearities in the system behavior and transient strengthening are taken into account. These additional aspects of a localizing system are needed to stabilize localization and probably limit the divergent growth rate of the localization instability to a finite value. Therefore, applications to the Earth need not consider the doublet opening and can replace it conceptually with a single peak at the preferred wavelength of the instability $k^j_0$ (Eq. 20).

4.3. Effect of substratum viscosity

The buckling instability takes its origin in the viscosity contrast across the interfaces of the model [Johnson and Fletcher, 1994]. Therefore, the growth rate of the buckling instability increases with the viscosity contrast. Buckling requires that the layer be stronger than the substratum.

The localization instability, on the other hand, is linked to a resonance that is internal to the layer with negative stress exponent. Therefore, it can grow even if the viscosity of the half-space is higher than the viscosity of the layer. However, in this case, the spectrum is offset by one half of the characteristic wavelength (Fig. 7). When the substrate is more viscous than the layer, the doublets are located at

$$k^j_0 = (j + 1/2) K_L, \quad j \in \mathbb{Z}. \quad (21)$$

Physically, the offset is required because the viscous substrate cannot follow the localized deformation. At wavenumbers halfway between actual resonances, an incipient shear zone interacts with a negative image of itself. Therefore, these shear zones interact destructively at the interface, which is required by the stronger substratum. When the viscosity of the substrate is small, it provides no resistance to the localized deformation and the preferred wavenumber is exactly at the resonance.

The position of the doublet changes continuously from Eq. 20 to Eq. 21 as the substrate viscosity increases (Fig. 7). In general, we write

$$k^j_B = (j + a_L) K_L, \quad j \in \mathbb{Z}, \quad (22)$$

with $a_L$ an empirical number called the spectrum offset, and $K_L$ the wavenumber scale defined in Eq. 19. The spectrum offset is 0 if $\eta_2/\eta_1 \ll 1$, $a_L = 1/4$ if $\eta_2/\eta_1 \approx 1$, and $a_L = 1/2$ if $\eta_2/\eta_1 \gg 1$.

For generality, we also define a spectrum offset $a_B$ for the buckling instability

$$k^j_B = (j + 1/2 + a_B) K_{B/N}, \quad j \in \mathbb{Z}, \quad (23)$$

with $K_{B/N}$ the wavenumber scale defined in Eq. 17. Eqs. 22 and 23 are written so that $a_B = a_L = 0$ for models where both the layer and the substrate have constant viscosity and the layer is much stronger than the substrate (Fig. 2). This will change when depth-dependent viscosity is considered in the model layers.

4.4. Effect of model density

A density contrast at the surface of the model reduces the growth rate of the buckling instability and increases its preferred wavenumber because of the restoring force on the growing surface topography [Zuber, 1987; Martinod and Molnar, 1995; Neumann and Zuber, 1995]. If $n_e > 0$, the growth rate is particularly reduced at small wavenumber. If $1/n_e \to 0^+$, the density-induced reduction of the growth rate at small $k$ can result in the maximum growth rate being at a resonance with $j \geq 1$ [Neumann and Zuber, 1995]. In addition, there is no growth of the buckling mode over half of the wavenumbers, from $jK_{B/N}$ to $(j + 1/2)K_{B/N}$ at small $j$. This produces a spectral offset in Eq. 23 up to $a_B = 1/4$ for $j = 1$ and large density of the model. The wavelength of buckling in the Central Indian Ocean may be larger to the north of the basin where the surface density contrast is reduced by the large sediment supply of the Bengal fan [Zuber, 1987].

If the layer has localizing properties $(1/n_e < 0)$, the model density influences the buckling part of the growth spectrum in the same manner as described above, except that growth rate is enhanced near the divergent doublets of the localizing instability. The effect is particularly pronounced at small wavenumber and increases with the density of the model (Fig. 8). However, scaling to Earth conditions indicates that the normalized density $pH/\eta_1\varepsilon_{11}$ should be of order 1 to 30 [Zuber et al., 1986], which is small. For these values, the density has little effect on the localization instability, although it does bring a spectral offset up to $a_B = 1/4$ for the longest preferred wavelength of the buckling instability $(j = 0)$.

5. Models with depth-dependent viscosity

The models considered in the previous section are only crude approximations to the Earth’s structure. The layer represents the brittle crust and upper mantle, and the half-space represents the hotter ductile rocks leading to the asthenosphere. The interface between the layer and the half-space corresponds to the brittle–ductile transition.

Unlike the models in the previous section, the strength profile of the Earth is continuous across the transition from brittle to ductile behavior. In idealized representations of the strength profile of the lithosphere, the brittle–ductile transition occurs at a particular depth where the brittle and ductile strengths of rocks are identical [Brace and Kohlstedt, 1980], or is distributed over a depth range where the brittle and ductile rock strengths are comparable [Kirby, 1980; Kohlstedt et al., 1995]. In any case, the strength and therefore the apparent viscosity of the lithosphere varies continuously with depth within each layer, due to the pressure dependence of rock strength in the brittle regime [Scholz, 1990], and the temperature dependence of creep in the ductile regime [Kohlstedt et al., 1995].

In the following sections, we change progressively the simple strength profile of the previous section to a more realistic strength profile similar to Fig. 10b, except for varying $r_2$ and $n_3 = -10$. Negative values of $r_2$ indicate that the substrate viscosity increases exponentially with depth.

Figure 9. Map of growth rate as a function of decay length of substrate viscosity and perturbation wavenumber. Strength profile similar to Fig. 10b, except for varying $r_2$ and $n_3 = -10$. Negative values of $r_2$ indicate that the substrate viscosity increases exponentially with depth.
profile, keeping track of the preferred wavelengths of buckling and localizing instabilities, as well as their growth rate. Our goal is to derive a simple prediction of the preferred wavelength of the localization instability relevant for a layer of rock undergoing a brittle–ductile transition at a specific depth with a strength profile similar to the Earth’s. The effect of having the brittle–ductile transition distributed over a finite depth range or density contrasts within the lithosphere will be considered elsewhere.

The wavenumber scaling of the instability is still valid when depth–dependent viscosity is considered. Therefore, we use Eq. 22 and 23 to describe the preferred wavenumbers of each instability. However, the value of the spectral offset parameter is empirically determined for each type of viscosity profile. The viscosity is scaled to 1 at the bottom of the brittle layer. We use \( n_e = -10 \) as an illustration for localizing behavior. In that case, \( K_L \approx 10 \).

### 5.1. Exponential decay of viscosity in the substrate

Because the stream function for a layer of exponential viscosity profile is proportional to \( \exp[k(z + \alpha z)] \) (Eq. 2 and 13), a perturbation with wavelength \( \lambda \) penetrates into a layer to a depth \( z_d = \lambda/\text{Im}(\alpha) \). Hence, it senses a viscosity averaged over \( z_d \). Therefore, a buckling instability can grow even if the strength profile is continuous at the boundary between the layer and the substrate if the viscosity of the substrate decreases exponentially with depth [Fletcher and Hallet, 1983; Zuber and Parmentier, 1986]. In fact, most applications of buckling or necking to the tectonics of terrestrial planets have used a strength profile made of a layer of uniform viscosity lying over a substrate with viscosity profile

\[
\eta = \eta_e \exp(r z),
\]

A value of \( r \approx 10 \) is often appropriate [Fletcher and Hallet, 1983; Zuber and Parmentier, 1986].

There are two differences between the growth spectrum of a layer lying over a substrate with exponentially–decaying viscosity and the previous case of a constant–viscosity half–space, even if the layer is plastic \((1/n_1 \to 0, \text{buckling instability only}, \text{Fig. 10})\). First, the envelope of the growth spectrum decreases at high wavenumber. This is because the short wavelength senses only the top of the substrate, which has higher viscosity than deeper levels, and therefore smaller viscosity contrast [Ricard and Froidevaux, 1986; Zuber and Parmentier, 1986; Neumann and Zuber, 1995]. Second, the instability grows only over one half of the range of wavenumbers, between \( j K_{B/N} \) and \((j + 1/2) K_{B/N}, j \in \mathbb{Z} \). Hence, the preferred wavenumbers of buckling become

\[
k_j^B = (j + 1/4) K_{B/N}, \quad j \in \mathbb{Z},
\]

or, using Eq. 23, the spectrum offset \( a_B \) is 1/4 for this strength profile.

When the layer is undergoing localization \((n_1 < 0)\), the growth spectrum is described as the superposition of a buckling–like spectrum and a sequence of divergent doublets representing the localization instability (Fig. 10), as in the model with constant viscosity layers (Fig. 2). At the smallest wavenumbers, the substrate appears very weak, and the spectral offset \( a_L \sim 0 \) for \( j = 0 \). At larger
wavenumbers, however, the substrate viscosity is similar to that of the layer, so that the spectral offset $a_L = 1/4$ (see below Eq. 22).

In summary, the preferred wavelength of localization follows Eq. 22 with approximately

$$a_L = \begin{cases} 
0, & \text{if } j = 0, \\
1/4, & \text{if } j > 0.
\end{cases}$$

(26)

Fig. 9 shows how the growth spectrum varies as a function of the decay–depth of the viscosity profile. Note how the buckling instability vanishes when $r < 0$ (viscosity of the substrate increasing exponentially with depth), whereas the localization instability is still present. However, the substrate is now stronger than the layer, so that $a_L = 1/2$ at small $j$.

5.2. Depth–increasing strength of the layer

As the layer corresponds to rocks undergoing brittle deformation, its strength should increase with depth. We first consider models in which the viscosity of the layer increases exponentially with depth, which is mathematically more tractable, and then the more realistic case of a viscosity increasing linearly with depth in the layer. In both cases, the viscosity is $\eta_1 = 1$ at the base of the layer. The substrate has a constant viscosity of $\eta_2 = 0.1$ and a non–Newtonian behavior with $n_2 = 3$.

5.2.1. Exponential viscosity profile. Having an exponential viscosity profile in the layer reduces its apparent viscosity. Accordingly, the growth rate of the buckling instability is reduced compared to the constant viscosity case but its preferred wavelength is unchanged (Fig. 12). When the material in the layer is pseudo–plastic ($1/n_1 \approx 0.1$) the envelope of the growth spectrum does not depend on wavenumber because the depth of penetration into the layer of the perturbation is infinite ($\text{Im}(\alpha) \to 0$): the whole layer is sampled at all wavelengths. When the layer is localizing ($n_1 < 0$, Fig. 12), the preferred wavelength of the localization instability is offset by $1/4$ of the characteristic scale $K_L$. Interestingly, the amplitude of the buckling mode decreases at the smallest wavenumbers if $1/n_1 < 0$ and the layer viscosity increases with depth (Fig. 12).

A complication arises because the mode slope $\alpha$ depends on the decay parameter of the viscosity profile (Eq. 14). This changes the resonant wavenumbers (Fig. 11) and therefore the wavenumber scale of instabilities $K_L$ and $K_{B/N}$. Although the localization instability still follows the resonance (Fig. 13), there is no longer an analytical expression for $K_L$ or $K_{B/N}$. However, Eq. 19 is approximately valid when $r \approx 0$, which is the case for realistic viscosity profiles. Therefore, the preferred wavelengths of localization are given approximately by 22 with

$$a_L = 1/4, \quad j \in \mathbb{Z}.$$  

(27)

Note that the first localization doublet ($j = 0$) is wider than for other strength profiles.

The long wavelength limit $rk > 2\sqrt{2 + \sqrt{5}}$, beyond which not all the solutions of $\alpha$ are real even if $1/n_1 < 0$, does not prevent the growth of a localization instability. This is because for $1/n_1$.
sufficiently negative, one value of the mode slope, \( \alpha \), is pure imaginary: there are still two values of \( \text{Re}(\alpha) \), one being zero, and the resonance depicted in Fig. 6b is still defined.

5.2.2. Linear viscosity profile. Although an exponential viscosity profile is only a poor approximation of the linear increase of strength with depth expected in the brittle layer from friction laws, there is little difference between the results of the previous section and the growth spectra obtained with the linear law. The amplitude of the buckling mode is reduced, and the preferred wavelength of the localization instability is offset by \( 1/\lambda \) of the wavelength scale \( K_L \) at small wavelengths. In addition, the resonance wavelength is close to the analytical value of Eq. 19 obtained for a constant viscosity layer. This is because the average strength of the layer is limited to one half of its maximum value when it increases linearly with depth. The decay parameter \( r \) for exponential viscosity profile that produces the same characteristics is small. Indeed, the growth spectrum for a layer with linear viscosity profile is closest to the case \( r = -2 \) with an exponential viscosity profile. For these values the resonant wavenumbers cannot be differentiated from the limit \( r = 0 \).

6. Discussion

6.1. Putting it all together: Growth spectrum for realistic strength profiles

A realistic strength profile for application to tectonics has a plastic or brittle layer with strength increasing linearly with depth, followed by a layer of half–space of ductile material with viscosity decreasing with depth. We learned from the previous sections that there are two superposed instabilities for a plastic or localizing layer overlying a half–space: the buckling instability that results in broad undulation of the layer as a whole when that layer is stronger than the substratum averaged over a wavelength–dependent penetration depth, and the localizing instability that produces regularly–spaced faults or shear zones. Reintroducing \( H \), the thickness of the brittle layer, as a length scale in Eq. 17, 19, 22, and 23, these instabilities grow preferentially at the wavenumbers

\[
\begin{align*}
K_B & = (j + 1/2 - a_B) K_{B/N}, \\
K_L & = (j + a_L) K_L,
\end{align*}
\]

with \( j \) an integer, \( a_B \) and \( a_L \) spectral offsets that depend on the strength profile, and \( K_{B/N} \) and \( K_L \) wavenumber scales that correspond to resonances in the brittle layer and are given by

\[
\begin{align*}
K_{B/N} &= \pi (1 - 1/n_1)^{-1/2}, \\
K_L &= \pi (-1/n_1)^{-1/2}.
\end{align*}
\]

Depth–increasing viscosity in the layer, depth–decreasing viscosity in the half–space, and buoyancy forces all decrease the growth rate of the buckling instability (Fig. 10 and 12). Hence, the buckling instability shows only modest growth rates for the most realistic strength profile used in this study (Fig. 14). Furthermore, the exponentially–decaying viscosity in the substrate suppresses the instability over half of the wavenumber range (Fig. 10), and a surface density contrast cancels the instability over the other half of the

![Figure 14. a) Growth spectrum for a layer in which the viscosity increasing linearly with depth overlying a half–space in which the viscosity decays exponentially with depth, with \( \rho = 0, n_2 = 3 \). Solid line: \( n_1 = -10 \); dashed line: \( n_1 = 10^6 \). b) Viscosity profile.](image)

![Figure 15. Map of growth rate as a function of the model density \( \rho g / \eta_i \epsilon_{ii} \) and wavenumber. Viscosity profile similar to Fig. 14b. Lighter tone indicates high growth rate, with the contours indicated the color bar. As the model density increases, the buckling mode vanishes.](image)
wavenumber range (Fig. 8). It follows that buckling is not a likely expression of shortening in a layered lithosphere (Fig. 15) unless the surface density contrast is reduced. Indeed, natural examples of lithospheric-scale buckling are associated with deformation under a heavy fluid, which reduces the surface density contrast. In the Central Indian Ocean, this fluid stands for the sediments from the Bengal fan [Zuber, 1987; Martinod and Molnar, 1995]. Many other regions in which buckling has been documented are under sedimentary basins [Burov et al., 1993; Cloetingh et al., 1999]. On Venus, the ridge belts grew in the same time that basaltic flood plains were emplaced [Zuber and Parmentier, 1990; Stewart and Head, 2000]. If buckling does grow, the relevant spectral offset is

\[ 0 < a_{B/N} < 1/4. \]  

Alternatively, it can be argued that including more realistic behavior would help buckling even in present of a relatively high surface density contrast. Schmalholz et al. [2002] show that including that consideration of visco-elastic behavior influence that manner that their folding solution depends on surface density contrast. A dynamic surface redistribution condition [Biot, 1961; Beaumont et al.,

Figure 16. a) Growth spectrum for a layer of uniform viscosity \( \eta_1 \) overlying a half-space of uniform viscosity \( \eta_2 = \eta_1/10 \), with \( \rho = 0 \), \( n_2 = 3 \) undergoing horizontal extension. Solid line: \( n_1 = -10 \); dashed line: \( n_1 = 10^6 \); dotted line: \( n_1 = 100 \). b) Viscosity profile.

Figure 17. Deformation fields corresponding to the growth spectra in Fig. 16, shaded as a function of strain rate. a) \( n_1 = 10^6 \); b) \( n_1 = -10 \). Models undergoing extension. Construction otherwise similar to Fig. 3.
Neither depth–dependent viscosity nor surface density contrasts reduce the growth rate of the localization instability. Depth–dependent viscosity in the layer and in the substrate each offsets the preferred wavelength by about $1/4 \ K_B/N$. The density of the model also increases the spectral offset (Fig. 15). All things considered, the spectral offset for a realistic viscosity profile is

$$1/4 < a_L < 1/2.$$  \hspace{1cm} (31)

A map of growth rate similar to Fig. 5 but for a realistic strength profile is presented in Fig. 18. It shows how the wavenumber of the localization instability varies with the effective stress exponent of the layer. The localization instability is seen to follow the resonant wavenumbers, $K_L$. The buckling mode of deformation all but vanishes when depth–dependent viscosity is taken into account. It is visible only near the divergent doublets of the localization instability.

### 6.2. A note about extension

Although the previous sections assumed horizontal shortening of the model, the formalism is equally valid for horizontal extension, for which $\tilde{2}_{xx} > 0$. However, the kinematic contribution of the primary flow to the growth of interface perturbations (Eq. 3) has the tendency to erase the imposed perturbation under horizontal extension [Smith, 1975]. Hence, only wavelengths with $Q > 1$ can be observed.

We present in Fig. 16 the growth spectra for a pseudo–plastic or brittle layer of uniform viscosity over a weaker half–space under horizontal extension. The corresponding deformation fields are plotted in Fig. 17. The spectra are rather similar to the shortening case (Fig. 2). In particular, the wavenumbers of the growth rate maxima for a pseudo–plastic layer ($n_e = 10^5$) and of the divergent doublets for the localizing layer ($n_e = 10$) are similar to the shortening case. The major difference between horizontal extension and shortening is the shape of the most unstable deformation mode over the whole model: the pseudo–plastic layer is necking under extension rather than buckling (Fig. 17a). The localization instability gives rise to regularly–spaced localized shear zones (Fig. 17b).

In presence of depth–dependent viscosity and density, the approach of a spectral offset (Eq. 23 and 22) is still valid. However, the spectral offset for the necking instability is $-1/4 < a_B < 0$ if the viscosity of the layer increases linearly with depth and the viscosity of the half–space decreases exponentially with depth. Hence, the wavelength of necking is generally smaller than the wavelength of buckling. As was observed in the shortening case, depth–dependent viscosity and model density conspire to reduce the range of wavelengths where growth of the necking instability is possible under horizontal extension, diminishing the likelihood that necking be observed in the tectonic record, unless the surface density contrast is small. Necking has been observed in nature, most prominently in the Basin–and–Range province [Fletcher and Hallet, 1983; Zuber et al., 1986; Ricard and Froidevaux, 1986] and plays an important role in rifting [Zuber and Parmentier, 1986; Lin and Parmentier, 1990]. Grooved terrain on Jupiter’s satellite Ganymede may also be formed by necking [Dombard and McKinnon, 2001]. The spectral offset of the localization instability in extension is the same as in compression, $1/4 < a_L < 1/2$.

### 6.3. Comparison with numerical studies

In early studies of fault patterns, faults were either a posteriori markers of deformation or a priori boundary conditions. In neither case is faulting a dynamic feature of the models or can the self–consistent pattern of faulting be determined. The instability of the localization process, which is expressed in our study by the fact that the effective stress exponent is negative, presents many analytical and numerical challenges. However, recent numerical methods have been able to present a continuum approach to localization, from microscopic scale [Hobbs and Ord, 1989; Poliakov et al., 1994] to global scale [Bercovici, 1993; Tackley, 2000]. With numerical models, it is possible to go beyond the instantaneous patterns of faulting explored in this paper to study how faulting evolves over time [Sornette and Vanneste, 1996; McKinnon and Garri edo de la Barra, 1998; Buck et al., 1999; Cowie et al., 2000; Huismans and Beaumont, 2002]. Our analysis provides a physical insight into the origin of the macroscale fault patterns.

Using the numerical method FLAC [Cundall, 1989], Poliakov and coworkers explored the patterns of faulting in elastic–visco–plastic models for different tectonic environments [Buck and Poliakov, 1998; Gerbault et al., 1998, 1999; Cloetingh et al., 1999; Lavie et al., 2000]. Even in the absence of explicit weakening, an elastic–plastic rheology is characterized by a negative stress exponent, or dynamic strain–weakening, because the strain and stress increments upon failure are not collinear [Montési and Zuber, 2002]. Localization by strain–weakening may behave differently from the strain–rate–weakening used in our paper. However, the effective exponent provides a unifying measure of localization, and it is relevant to compare the numerical results in presence of strain–weakening to our model, provided that we use $-0.1 < n_e < -0.01$, as appropriate for localization in elastic–plastic materials [Montési and Zuber, 2002]. The fault spacing predicted by our analysis ($0.4 < \lambda/H < 2.5$) is consistent with the spacing observed in numerical models. The localization instability (adapted for strain–weakening) is a likely origin of the fault pattern observed in numerical models.

Explicit strain–softening was shown to enhance faulting in elastic–visco–plastic models and to increase the fault spacing [Gerbault, 1999]. This is again consistent with the localization instability, which predicts larger fault spacings for more efficient localization. However, if the weakening is too strong, another transition

---

**Figure 18.** Map of growth rate as a function of effective stress exponent of the layer and perturbation wavenumber. Lighter tone indicates high growth rate, with the contours indicated on the color bar. Viscosity profile identical to Fig. 14b. Note how the buckling instability is reduced compared to Fig. 5.
occurs and a single fault develops in numerical models [Lavier et al., 2000]. The resulting deformation pattern is sometimes asymmetric [Lavier et al., 1999; Huismans and Beaumont, 2002]. Frederiksen and Braun [2001] also observed localization on a single fault and its conjugate in their models that include strain-softening of the viscous, rather than the plastic rheology. They also showed that the fault intensity depends on the rate of weakening, consistent with localization being controlled by the effective stress exponent rather than only the amount of weakening. Localization to a single fault is not predicted by our model. It may be due to second order or finite strain effects that we do not address yet. Sornette and Vanneste [1996] and Cowie et al. [2000] also report on localization of strain over a single fault upon finite deformation. Rather than following an elastic–plastic rheology, their models are elastic, with fault slip accumulating when a yield criterion is verified. The initial pattern of faulting is dominated by the strong prescribed heterogeneity in these models, which prevents a characteristic length–scale from developing. As slip on a fault enhances the stress in the vicinity of the fault tip, the tendency to failure of faults in that region is enhanced. After finite slip, the fault pattern may localize because of the interaction between neighboring faults [Sornette and Vanneste, 1996; Cowie et al., 2000]. The interaction between several active faults may be described with a negative effective stress exponent. Future improvement of our model, in particular including a higher-order or time-dependent analysis may address this later instability of fault pattern.

The numerical studies closest to our study consider strain–rate softening in visco–plastic models [Neumann and Zuber, 1995; Montési and Zuber, 1997, 1999]. They produce regularly–spaced faults superposed on either buckling or necking. Numerical results suggest that faults are mostly active in the antilines of lithospheric–scale folds [Montési and Zuber, 1997, 1999] or the necks of lithospheric scale boudins [Neumann and Zuber, 1995]. This interaction between the buckling/necking and faulting deformation fields is not predicted in our analysis, for which faults are present everywhere (Fig. 3 and 17), but may result from an higher-order interaction between the buckling/necking and localization instabilities. Numerical results indicate several faults in the growing anticlines or necks. Their spacing is consistent with the prediction of our model, showing again a control of the fault pattern by the localization instability. As deformation proceeds, some faults cease their activity, and others replace them [Montési and Zuber, 1997, 1999]. Switches in fault patterns are discrete in time. They reuse recently active faults, with new faults formed in the front of the existing deformation zone, separated from it by the same space as within the deformation zone. Thus, the localization–instability–controlled fault spacing is prominent at finite strain. Propagation of a deformation front with characteristic fault spacing is also observed in elastic–plastic models [Hardacre et al., 2001], but to our knowledge, the fault spacing in that case has not been studied.

In summary, fault sets produced by numerical models may show a preferred spacing that is consistent with the prediction of the localization instability. However, a high level of initial heterogeneity can prevent a regular spacing to form [Sornette and Vanneste, 1996]. With finite deformation, the fault pattern may collapse on a single fault [Sornette and Vanneste, 1996; Cowie et al., 2000; Lavier et al., 2000; Frederiksen and Braun, 2001; Huismans and Beaumont, 2002] through a process that we cannot address here. All the models undergoing horizontal shortening show regularly–spaced fault sets even at finite strain [Gerbault et al., 1999; Montési and Zuber, 1999]. Accordingly, compressive orogens often display a propagating deformation front with regularly–spaced faults [e.g., Hoffman et al., 1988; Meyer et al., 1998]. However, the importance of sub–horizontal decollements for this behavior remains to be evaluated.

Other numerical studies focused on the localization of shear within a fault gouge [Morgan and Boettcher, 1999; Place and Mora, 2000; Wang et al., 2000] or the spatio–temporal localization of slip over a seismogenic fault [Ben Ziel and Rice, 1995; Miller and Olggaard, 1997; Lapusta et al., 2000; Madariaga and Olsen, 2000]. As our study does not resolve the temporal evolution of localization and assumes a different geometry than that relevant for seismogenic and fault gouge processes, we cannot compare our work and these studies. Similarly, our analysis cannot be applied directly to regularly–spaced fault sets in strike–slip environments [Bourne et al., 1998; Roy and Royden, 2000]. However, the use of a negative stress exponent to build simple models of localization can be adapted to these problems. We hope that future developments of our model will address these different geometries as well as the interaction between the localization and buckling/necking instabilities.

7. Conclusions

We have presented new solutions of the perturbation analysis of mechanically layered models of the lithosphere undergoing shortening in which a brittle layer lies over a ductile substrate. In addition to the classically–recognized buckling instability, the layer may undergo a localization instability that results in regularly–spaced faults or shear zones. Localization is possible when the effective stress exponent of the brittle layer, a general measure of the mechanical response of the material to local perturbations, is negative [Montési and Zuber, 2002]. However, localization of deformation produces incipient shear zones in which the deformation field tries to develop a discontinuity that is not compatible with coupling with a ductile substrate. Therefore, shear zones cannot develop unless there is a resonance between several incipient shear zones. This resonance is the basis for a scaling wavenumber, $K_L$, that controls the wavelengths of instability. The resonance that is at the origin of $K_L$ exists only if the effective stress exponent is negative (Eq. 19). The actual wavelength of the instability is linked to $K_L$ by Eq. 22 which also includes a “spectral offset” parameter, $a_L$, that is calibrated empirically in function of the strength profile in the model. For a realistic profile where the strength of the brittle layer increases linearly with depth and the strength of the substrate decreases with depth, $1/4 < a_L < 1/2$. Similar principles are used to describe the buckling instability except that the scaling wavenumber, $K_B/N$, is rooted in a different resonance that does not require a negative effective stress exponent (Eq. 17), and that the spectral offset $a_B$ (Eq. 23) is between 0 and 1/4 in compression and between $-1/4$ and 0 in extension. Model density has only a minor effect on the localization instability. Buckling is much reduced when depth–dependence viscosity is included and is a likely expression of tectonic deformation only if the surface density contrast is reduced, for instance because of a high erosion or sedimentation rate.

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References


References


Davies, R. K., Models and observations of faulting and folding in layered rocks, Ph.D., Texas A&M University, 1990.


Fletcher, R. C., Effects of pressure solution and fluid migration on initiation of shear zones and faults, Tectonophysics, 295, 139–165, 1998.


