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Crustal remanence in an internally magnetized non-uniform shell: a possible source for Mercury's magnetic field?

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Abstract

We consider the magnetic field of a shell uniformly magnetized by an internal dipole that is subsequently removed. The Gauss coefficients of the resulting field are given in terms of the spherical harmonic coefficients of the shell thickness. This general solution can easily be reduced to common special cases by superposition. For a shell of constant thickness the external field vanishes (by Runcorn's theorem). For shell thickness variations caused by a laterally varying temperature field, the resulting magnetic moments are appreciably greater than the previously published correction due to rotational flattening. If the crust of Mercury contains rocks capable of sustaining high specific magnetizations, orif Mercury had a now-extinct dynamo field which was more intense than the Earth's present core field, then the Mariner 10 observations of Mercury's magnetic field are consistent in magnitude and geometry with the predictions of this model. For such a scenario, the requirement of a fractionally large molten outer core supporting a dynamo at present would be relaxed. The origin of Mercury's magnetic field will be addressed with measurements to be made by NASA's MESSENGER spacecraft.

Keywords: Mercury; magnetism; Runcorn's Theorem

1. Introduction

The internal state of Mercury is puzzling. The simplest thermal evolution models for Mercury (e.g., [1]) predict that, because of its small size, the planet should cool rapidly and its initially molten core should now be fully or nearly solid. The discovery by Mariner 10 of Mercury's global magnetic field caused a dramatic shift in thinking, and the field is now often ascribed to dynamo generation by convection in a molten outer core of substantial thickness [2,3]. This unexpected observation and its interpretation led to the emergence of models that attempt to keep Mercury's interior hot and the core largely molten. These conditions may be achieved by several means; two popular ideas are the inclusion of sulfur in the outer core, greatly reducing the melting point due to the low eutectic temperature of Fe–FeS, and the suppression of mantle convection, reducing the efficiency of heat loss. The arguments for

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rejecting the alternative hypothesis, that the field is crustal in origin [4–8], were severalfold. First, intensely magnetized rocks were not believed to be present in the natural environments of the terrestrial planets. However, the Mars Global Surveyor (MGS) mission to Mars has provided a striking demonstration that materials with specific magnetizations of ∼20 A/m are in fact present [9,10]. Laboratory experiments further support the possibility that single- and multi-domain thermoremanent magnetization (TRM) in some minerals is significant [11], although the required oxidized minerals (e.g., hematite, magnetite) might not be expected in the likely water-poor crust of Mercury. Other magnetic minerals (e.g., pyrrhotite) can be considered for both Mars [12] and Mercury. Secondly, an elegant theorem by Runcorn [13,14], proving that a uniform shell magnetized by an internal source subsequently removed has no external field, argues that even if magnetic carriers are available in Mercury's crust, they cannot account for the observed field if they are distributed uniformly. Breaking the symmetry requisite to Runcorn's theorem is the subject of this paper.

Inhomogeneities in planetary crusts arise from a variety of mechanisms, such as impact cratering, tectonics, and magmatism. In part due to Mercury's spin–orbit coupling, by which the planet spins three times for every two revolutions around the Sun, mean surface temperatures are strongly dependent on position. For a constant, uniform, conductive thermal gradient within the lithosphere, this spatial dependence will result in variations in the depth to the Curie temperature T_c, above which no TRM is possible for any given magnetic carrier. Here, we investigate how such variations in the thickness of the layer that is available to be magnetized might be responsible for external magnetic fields. The following sections provide a general solution to the variable layer thickness problem, demonstrate some special cases that are easily obtained from it, and apply the formulation to Mercury. Our aim is not to dispute that Mercury's magnetic field may indeed originate in the core, but rather to reexamine the often dismissed possibility that it originates in the crust.

2. Magnetostatics

We consider the magnetic field of a homogeneous shell, magnetized by an internal dipolar field aligned with the spin (z) axis that is subsequently removed. We seek the solution for a shell with one spherical boundary and one arbitrary boundary. Arbitrarily shaped shells can then be considered by superposition of the solution for such simple shells. For concreteness, we choose the inner shell boundary to be spherical, and the outer boundary is kept arbitrary (Fig. 1).

In the absence of any free currents and time-dependent electric fields, the magnetic field $\mathbf{H}$ outside a presumed, now extinct dynamo, satisfies:

$$\nabla \times \mathbf{H} = 0 \quad (1)$$

Hence, the field can be written in terms of a potential:

$$\mathbf{H} = -\nabla \Phi_p \quad (2)$$

where in spherical coordinates the dipole potential at a point (r, θ, ϕ) is of the form:

$$\Phi_p = \frac{p}{4\pi} \frac{\cos\theta}{r^2} \quad (3)$$

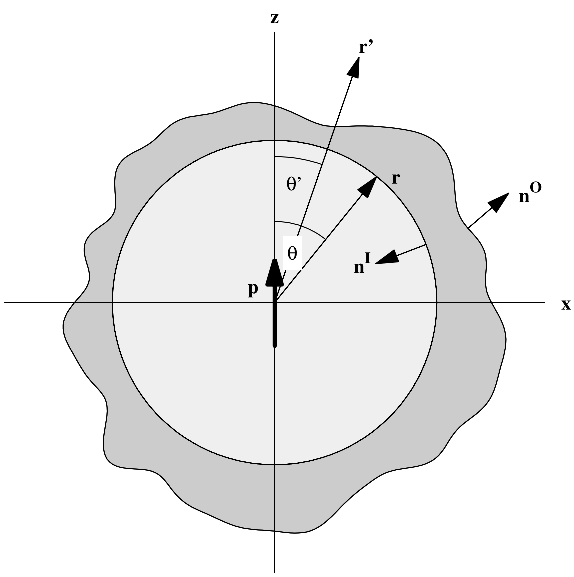

Fig. 1. Schematic illustration of a shell with an arbitrary outer boundary and a spherical inner boundary. The magnetizing dipole p is shown at the origin.

where p is the dipole moment. For a linear medium, the magnetization is:

$$\mathbf{M}(\mathbf{r}) = c\mathbf{H} = \frac{pc}{4\pi}\left(\frac{2\cos\theta}{r^3}\hat{r} + \frac{\sin\theta}{r^3}\hat{\theta}\right) \tag{4}$$

where $(\hat{r}, \hat{\theta}, \hat{\phi})$ are the unit vectors in spherical coordinates. While similar in form to the susceptibility, the constant c actually measures the ratio of the remanent magnetization to the applied field after the latter vanishes. The scalar potential $\Phi M(\mathbf{r}')$ resulting from this magnetization distribution $\mathbf{M(r)}$ that remains after the applied field $\mathbf{H}$ is removed is ([15], p. 197, in SI units):

$$\Phi_{\mathrm{M}}(\mathbf{r}') = -\frac{1}{4\pi}\int \frac{\nabla\cdot\mathbf{M}(\mathbf{r})}{|\mathbf{r}-\mathbf{r}'|}\mathrm{d}v + \frac{1}{4\pi}\int \frac{\mathbf{M}(\mathbf{r})\cdot\hat{\mathbf{n}}(\mathbf{r})}{|\mathbf{r}-\mathbf{r}'|}\mathrm{d}s \tag{5}$$

All integrals are over the entire volume v or surface s, where $\hat{\mathbf{n}}$ is the unit normal to the surface. The first term vanishes, and the second term can be expanded in Legendre polynomials:

$$\frac{1}{|\mathbf{r}-\mathbf{r}'|} = \frac{1}{r'}\sum_{l=0}^{\infty}\left(\frac{r}{r'}\right)^l P_l(\cos\gamma),\ r'>r \tag{6}$$

where γ is the angle between $\mathbf{r}$ and $\mathbf{r}'$. The spherical harmonic addition formula is used to write:

$$P_l(\cos\gamma) = \frac{4\pi}{2l+1}\sum_{m=-l}^{l} Y^*_{lm}(\theta,\phi)\ Y_{lm}(\theta',\phi') \tag{7}$$

where Y_{lm} is the complex spherical harmonic function of degree l and order m, and * denotes complex conjugation. Unless otherwise specified, summations over l and m below will be from 0 to ∞ and from $-l$ to l, respectively. The harmonics are defined in terms of the associated Legendre functions [16]:

$$Y_{lm}(\theta,\phi) = (-1)^m\sqrt{\frac{(2l+1)}{4\pi}\frac{(l-m)!}{(l+m)!}}P_{lm}(\cos\theta)e^{im\phi} \tag{8}$$

and are normalized to 1:

$$\int Y^*_{lm}(\theta,\phi)Y_{l'm'}(\theta,\phi)\mathrm{d}\Omega = \delta_{ll'}\delta_{mm'} \tag{9}$$

where $\mathrm{d}\Omega = \sin\theta\ \mathrm{d}\theta\ \mathrm{d}\phi$ and δ_{ij} is the Kronecker delta symbol, which equals 1 for $i=j$ and 0 otherwise. It is often convenient to define the shell boundary parametrically, so that on the integration surface:

$$\mathbf{r} = r(\theta,\phi)\hat{r} \tag{10}$$

Using the shorthand definition $\partial_\theta = \partial/\partial\theta$, $\partial_\phi = \partial/\partial\phi$, and:

$$\nabla_1 = \hat{\theta}\partial_\theta + \hat{\phi}(\sin\theta)^{-1}\partial_\phi \tag{11}$$

the unit normal is:

$$\hat{\mathbf{n}}(\mathbf{r}) = \frac{\hat{r}-\frac{1}{r}\nabla_1 r}{|\hat{r}-\frac{1}{r}\nabla_1 r|} \tag{12}$$

and the area element can be written as:

$$\mathrm{d}s = \frac{r^2}{\hat{r}\cdot\hat{n}}\mathrm{d}\Omega = |\hat{r}-\frac{1}{r}\nabla_1 r|r^2\mathrm{d}\Omega \tag{13}$$

Substituting these expressions into Eq. 5, and referencing to a radius a interior to r', the potential is written as:

$$\Phi_{\mathrm{M}}(r',\theta',\phi') = \frac{a}{\mu_0}\sum_{l,m}\left(\frac{a}{r'}\right)^{l+1} g_{lm}\ Y_{lm}(\theta',\phi') \tag{14}$$

where $\mu_0 = 4\pi\times10^{-7}$ Tm/A. Each of the Gauss coefficients g_{lm} will have two contributions, one from the integral over the outer surface, $g_{lm}{}^{\mathrm{O}}$, and the other from the integral over the inner surface, $g_{lm}{}^{\mathrm{I}}$:

$$g_{lm} = g^{\mathrm{O}}_{lm} + g^{\mathrm{I}}_{lm} \tag{15}$$

The contribution from each of the boundaries is:

$$g^{\mathrm{O,I}}_{lm} = \pm\frac{\mu_0}{4\pi}\frac{1}{(2l+1)}\frac{cp}{a^3}\int_{\mathrm{O,I}}\left(2\cos\theta - \frac{\sin\theta}{r}\partial_\theta r\right)\left(\frac{r}{a}\right)^{l-1}Y^*_{lm}(\theta,\phi)\mathrm{d}\Omega \tag{16}$$

The negative sign is appropriate for the inner boundary due to the opposite direction of the unit normal vector. It is convenient to expand the shell topography in terms of spherical harmonics:

$$r^{\mathrm{O,I}}(\theta,\phi) = \sum_{l'm'} r^{\mathrm{O,I}}_{l'm'}Y_{l'm'}(\theta,\phi) \tag{17}$$

Table 1
Normalized potential coefficients g_{lm} due to a shell of arbitrary topography, r_{lm}, uniformly magnetized by an internal dipole of moment p

l	$m=0$	1	2	3	4
0	0				
1	$\frac{2}{\sqrt{15}}r_{2,0}$	$\frac{1}{\sqrt{5}}r_{2,1}$			
2	$\frac{2}{5\sqrt{15}}r_{1,0}+\frac{18}{5\sqrt{35}}r_{3,0}$	$\frac{1}{5\sqrt{5}}r_{1,1}+\frac{12}{5}\sqrt{\frac{2}{35}}r_{3,1}$	$\frac{6}{5\sqrt{7}}r_{3,2}$		
3	$\frac{6}{7\sqrt{35}}r_{2,0}+\frac{12}{7\sqrt{7}}r_{4,0}$	$\frac{4}{7}\sqrt{\frac{2}{35}}r_{2,1}+\frac{3}{7}\sqrt{\frac{15}{7}}r_{4,1}$	$\frac{2}{7\sqrt{7}}r_{2,2}+\frac{6}{7}\sqrt{\frac{3}{7}}r_{4,2}$	$\frac{3}{7}r_{4,3}$	
4	$\frac{4}{9\sqrt{7}}r_{3,0}+\frac{20}{3\sqrt{11}}r_{5,0}$	$\frac{1}{3}\sqrt{\frac{5}{21}}r_{3,1}+\frac{8}{3}\sqrt{\frac{2}{33}}r_{5,1}$	$\frac{2}{3\sqrt{21}}r_{3,2}+\frac{4}{3}\sqrt{\frac{7}{33}}r_{5,2}$	$\frac{1}{9}r_{3,3}+\frac{16}{9\sqrt{11}}r_{5,3}$	$\frac{4}{3\sqrt{11}}r_{5,4}$

The coefficients can be related to the conventional Gauss coefficients by Eq. 27. The quantity $(\mu_0 cp/4\pi a^3 r_{00})(r_{00}/a)^{l-1}$ has been factored out of each term. Real fields can be described by terms with non-negative m, but in general this table is valid when each index m is replaced by $-m$.

In inserting Eq. 17 into Eq. 16, consider first the terms that are not proportional to $r_{l'm'}$, where $l'\geq 1$. The contribution of the integral of these terms due to the inner and outer boundary integrals sums to:

$$\int((r^{\mathrm{O}}_{00}/a)^{l-1}-(r^{\mathrm{I}}_{00}/a)^{l-1})2\cos\theta\, Y^*_{lm}(\theta,\phi)\mathrm{d}\Omega \qquad (18)$$

This quantity always vanishes, due to the radial term for $l=1$, and due to the angular integral for $l>1$. Consider now the terms proportional to $r_{l'm'}$, where $l'\geq 1$. Since the inner boundary was taken to be spherical, only terms due to the outer shell remain, and so we drop the superscript and refer to that expansion as simply r_{lm} hereafter. Assuming that all but the first coefficients are small:

$$\frac{r_{l'm'}}{r_{00}}\ll 1,\ l'\geq 1 \qquad (19)$$

Expanding in terms of the small quantities:

$$\frac{1}{r}\partial_\theta r\approx\sum_{l'\geq 1,m'}\frac{r_{l'm'}}{r_{00}}\partial_\theta Y_{l'm'}(\theta,\phi) \qquad (20)$$

and making use of the identities:

$$\int Y_{l_1m_1}(\theta,\phi)Y_{l_2m_2}(\theta,\phi)Y^*_{lm}(\theta,\phi)\mathrm{d}\Omega = \sqrt{\frac{(2l_1+1)(2l_2+1)}{4\pi(2l+1)}}C^{l0}_{l_1,0,l_2,0}C^{lm}_{l_1,m_1,l_2,m_2} \qquad (21)$$

where $C^{lm}_{l_1,m_1,l_2,m_2}$ are Clebsch–Gordan coefficients ([16], § 27.9) and:

$$\partial_\theta Y_{lm}(\theta,\phi) = m\frac{\cos\theta}{\sin\theta}\,Y_{lm}(\theta,\phi)+\sqrt{L-M}\ Y_{l,m+1}(\theta,\phi)\mathrm{e}^{-i\phi} \qquad (22)$$

for the derivative of the spherical harmonic functions [16], gives finally:

$$g_{lm}=\sum_{l'=0}^{\infty}\sum_{m'=-l'}^{l'}\frac{\mu_0}{4\pi}\frac{cp}{a^3}\sqrt{\frac{(2l'+1)}{(2l+1)^3}}C^{l0}_{l',0,1,0}\frac{r_{l'm'}}{r_{00}}\left(\frac{r_{00}}{a}\right)^{l-1}\left((2l-2-m')C^{lm}_{l',m',1,0}-\sqrt{2(L'-M')}C^{lm}_{l',m'+1,1,-1}\right) \qquad (23)$$

where $L=l(l+1)$, $M=m(m+1)$, $L'=l'(l'+1)$, and $M'=m'(m'+1)$. Table 1 lists g_{lm} up to degree and order 4.

The Schmidt quasi-normalized real spherical harmonic functions $\hat{p}_{lm}$ that are conventional in geomagnetism can be used to decompose the potential:

$$\Phi_{\mathrm{M}}(r',\theta',\phi')=\frac{a}{\mu_0}\sum_{l=0}^{\infty}\sum_{m=0}^{l}\left(\frac{a}{r'}\right)^{l+1}\hat{p}_{lm}(\cos\theta')(\hat{g}_{lm}\cos m\phi'+\hat{h}_{lm}\sin m\phi') \qquad (24)$$

where:

$$\hat{p}_{lm}(\cos\theta')=\sqrt{(2-\delta_{m0})\frac{(l-m)!}{(l+m)!}}P_{lm}(\cos\phi') \qquad (25)$$

The real multipliers $\hat{g}_{lm}$ and $\hat{h}_{lm}$ are known as Gauss coefficients and are related to their complex counterparts by:

$$\begin{aligned}\hat{g}_{l0} &= \sqrt{\frac{2l+1}{4\pi}}g_{l0} \\ \hat{g}_{lm} &= (-1)^m\sqrt{\frac{2l+1}{2\pi}}\mathrm{Re}(g_{lm}) \\ \hat{h}_{lm} &= -(-1)^m\sqrt{\frac{2l+1}{2\pi}}\mathrm{Im}(g_{lm})\end{aligned} \quad (26)$$

where Re and Im extract, respectively, the real and imaginary parts of their arguments.

3. Special cases

We consider three special cases of the general solution provided by Eq. 23 and Table 1: a constant-thickness shell as in Runcorn's theorem, a shell with a generic degree-two flattening, and a possible shell appropriate for Mercury.

3.1. Runcorn's theorem

Runcorn's [13,14] assertion, that lacking any lateral variations in shell thickness the external field vanishes, is seen in the trivial case in which all $r_{l'm'} = 0$ for $l' > 0$. In this case, g_{lm} is identically zero.

3.2. Flattened shell

Suppose the outer boundary is given by:

$$r(\theta) = a(1+\eta P_2(\cos\theta)) = a\left(1+\eta\frac{3\cos^2\theta - 1}{2}\right) \quad (27)$$

where a is the radius, and η is the flattening parameter, small with respect to 1. It is easy to show that for this shell, flattened symmetrically about the dipole (z) axis, the only two non-zero Gauss coefficients are:

$$\hat{g}_{10} = \frac{\mu_0}{4\pi}\frac{cp\eta}{a^3}\left(\frac{2}{5}\right) \quad (28)$$

and

$$\hat{g}_{30} = \frac{\mu_0}{4\pi}\frac{cp\eta}{a^3}\left(\frac{6}{35}\right) \quad (29)$$

Their ratio is $\hat{g}_{30}/\hat{g}_{10} = 3/7$. The inducing dipole field potential has as its sole Gauss coefficient:

$$\hat{g}^p_{10} = \frac{\mu_0}{4\pi}\frac{p}{a^3} \quad (30)$$

Hence, the $l=1$ induced potential is reduced relative to the imposed potential by the factor $\hat{g}_{10}/\hat{g}^p_{10} = (2/5)c\eta$. Note that this ratio is independent of a.

A comparison can be made with the effect of a shell of constant thickness t and flattening η. This case can be considered as the difference between two shells with radii:

$$r_1 = a(1+\eta P_2(\theta)) \quad (31)$$

$$r_2 = a(1+\eta P_2(\theta)) + t \quad (32)$$

$$\approx a\left(1+\frac{t}{a}\right)\left[1+\eta\left(1-\frac{t}{a}\right)P_2(\theta) + O\left(\frac{t}{a}\right)^2\right] \quad (33)$$

By superposition, the resulting dipole moment will be in proportion to the sum of the $l=1$ Gauss coefficients:

$$M_z = \left(\frac{\hat{g}^{M2}_{10} - \hat{g}^{M1}_{10}}{\hat{g}^p_{10}}\right)p = -\frac{2}{5}cp\eta\frac{t}{a} \quad (34)$$

This agrees with the result by Runcorn [14]:

$$M_z = \frac{16\pi ctp}{15a}\eta' \quad (35)$$

as he normalized the P_2 such that his flattening $\eta' = -3\eta/2$, and he has an additional factor of 4π due to choice of units.

Comparing the two cases, it is clear that the flattened, constant-thickness shell has a field smaller than the flattened outer boundary shell by a factor of t/a, which can be of order 1/100.

3.3. Mercury

Consider the case in which the magnetized layer thickness varies as a result of variations in depth to a single Curie temperature, controlled by surface temperature and uniform heat flow. Since the insolation pattern should be symmetric about the equator, only terms with even l are non-zero. Furthermore, symmetry about longitudes 0° and 90° results in only cosine terms with even m. These constraints simplify Table 1 considerably, such that the only non-zero Gauss coefficients up to degree and order 4 are g_{10}, g_{30}, and g_{32}. An appropriately constrained spherical harmonic expansion of modeled near-surface temperatures [17], yields:

$$\{r_{0,0}, r_{2,0}, r_{2,2}, r_{4,0}, r_{4,2}\}\mathrm{d}T/\mathrm{d}z$$
$$= \{1230, -131, 81, -82, 26\}\ \mathrm{K}$$

Other parameters in the expression for g_{lm} must be estimated. Mercury's planetary radius is taken to be $a = 2440$ km. The thermal gradient near the surface is assumed to be $\mathrm{d}T/\mathrm{d}z = 10$ K/km. For these parameters, the Curie depth would vary by about 10 km. A steeper (shallower) gradient would increase (decrease) the magnitude of the shell thickness variation and hence of the externally observed field. If the magnetization was acquired from a dynamo generated in the core, then one possibility is to assume the field strength at Mercury's core–mantle boundary (CMB) was some factor f, of order unity, times its present value at Earth's CMB, on the grounds that the square of the core field is expected to scale with rotation rate, density, and inverse conductivity, but not core size [18]. This would be the case if Mercury's dipole moment was approximately $p = 9.2 \times 10^{21}$ Am^2, roughly an order of magnitude smaller than Earth's present moment. However, the dynamo could have been even more intense early in the history of both Earth and Mercury. The implicit assumption that the dynamo was constant during the cooling of the shell is likely to be an oversimplification in light of the fact of typical reversal timescales for the present Earth dynamo are shorter than the cooling time expected for such a shell. Combining these assumptions yields the following for the remanent field (written in terms of the often tabulated Schmidt-normalized Gauss coefficients):

$$\hat{g}_{10} = -85\ \mathrm{nT}\cdot fc$$
$$\hat{g}_{30} = -139\ \mathrm{nT}\cdot fc \qquad (37)$$
$$\hat{g}_{32} = 63\ \mathrm{nT}\cdot fc$$

Under these assumptions, the effect described may be important for explaining Mercury's field if the product of the parameters fc is near unity. For lunar basalts, the fraction of remanent magnetization is thought to correspond to $c \ll 1$ [13]. However, magnetic fields observed by MGS in Mars orbit are as large as $\sim$1500 nT [9,10] and are believed to originate in the crust. In terms of specific magnetization, such observed fields indicate values of at least 20 A/m [9,19]. If magnetized in a field comparable to the Earth's $B \approx 50\,000$ nT, or $H \approx 40$ A/m, these values imply c can indeed approach unity. Of course, the larger the value for f, the smaller the value for c can be to produce a measurable field.

If c is large, non-linear effects of self-magnetization may not be negligible. For example, the effect of the Earth's ellipticity has an appreciable contribution from terms that are non-linear in the susceptibility [20,21]. Combining a variable Curie temperature depth with this effect will require a more complicated model than is presented here, which would include, for example, the early thermal evolution of the crust.

The best estimates to date for the magnetic field of Mercury were obtained by Mariner 10 during Mercury encounters I and III [22]. The estimates for the dipole component $\hat{g}_{10}$ range between 227 and 350 nT. This range of estimates can be accounted for by attributing some of the observed power to higher moments (such as an axisymmetric quadrupole) [22]. The constraint on the dipole moment is accordingly imprecise.

The computed Gauss coefficients above demonstrate that the effect described may be relevant to explaining the nature of Mercury's magnetic field. However, their values should not be viewed as strict predictions. Rather, they are the consequences of an idealized crust containing none but the long-wavelength inhomogeneity due to present-day surface temperatures. A variety of processes in the past and present surely have affected the crustal structure, and hence, any remanent field.

4. Conclusions

1. A general solution to the problem of a magne-

tized shell of arbitrary thickness has been obtained. The Gauss coefficients of the resulting field are given in terms of the spherical harmonic coefficients of the shell thickness. This solution can easily be reduced to common special cases using superposition.

2. For a shell of variable thickness, such as might be inherited from a variable temperature field, the resulting magnetic moments are $\sim$100 greater than the previously published correction due to rotational flattening [14], the latter being suppressed relative to the former by a factor of t/a.
3. If Mercury's crust contains rocks capable of sustaining high specific magnetizations, or if Mercury had a now-extinct dynamo field which was more intense than Earth's present core field, then the observations of Mercury's present magnetic field are consistent in magnitude and geometry with the predictions of this model.
4. The question of whether Mercury's present magnetic field is generated in the crust will be answered rapidly upon the arrival of the MESSENGER spacecraft [23] to the planet, as its on-board magnetometer should detect asymmetries in the field if it is crustal. Predictions of the decay of the field's power spectrum differ depending on the assumed source. While it is possible, as shown here, for both the crust and the core to lead to low-order field components, a crustal field must possess local heterogeneities (due to structures such as impact craters), implying more power at shorter wavelengths, that is, a slower decay of the power spectrum. High-order components may be present in a core field, but they would be attenuated by upwards continuation to spacecraft altitude.

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