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A procedure for determining the nature of Mercury's core

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Abstract–We review the assertion that the precise measurement of the second degree gravitational harmonic coefficients, the obliquity, and the amplitude of the physical libration in longitude, C_{20}, C_{22}, θ, and ϕ_0, for Mercury are sufficient to determine whether or not Mercury has a molten core (Peale, 1976). The conditions for detecting the signature of the molten core are that such a core not follow the 88-day physical libration of the mantle induced by periodic solar torques, but that it does follow the 250 000-year precession of the spin axis that tracks the orbit precession within a Cassini spin state. These conditions are easily satisfied if the coupling between the liquid core and solid mantle is viscous in nature. The alternative coupling mechanisms of pressure forces on irregularities in the core–mantle boundary (CMB), gravitational torques between an axially asymmetric mantle and an assumed axially asymmetric solid inner core, and magnetic coupling between the conducting molten core and a conducting layer in the mantle at the CMB are shown for a reasonable range of assumptions not to frustrate the first condition while making the second condition more secure. Simulations have shown that the combination of spacecraft tracking and laser altimetry during the planned MESSENGER (MErcury Surface, Space ENvironment, GEochemistry, Ranging) orbiter mission to Mercury will determine C_{20}, C_{22}, and θ to better than 1% and ϕ_0 to better than 8%—sufficient precision to distinguish a molten core and constrain its size. The possible determination of the latter two parameters to 1% or less with Earth-based radar experiments and MESSENGER determination of C_{20} and C_{22} to 0.1% would lead to a maximum uncertainty in the ratio of the moment of inertia of the mantle to that of the whole planet, C_m/C, of ~2% with comparable precision in characterizing the extent of the molten core.

INTRODUCTION

The discovery of an intrinsic magnetic field for Mercury by the *Mariner 10* flyby (Ness *et al.*, 1974) and the preference for an internal dynamo as the source (Ness *et al.*, 1975) are the primary motivations for believing that Mercury has a core that is currently molten. However, modelers of the thermal history of the planet are not unanimous in their predictions of the current state of the core. From a model in which Mercury is fully differentiated, Siegfried and Solomon (1974) concluded that an initially molten, pure iron core would tend to cool and solidify over geologic timescales. Whether or not a given model with differentiation has a molten core that persists to the present depends critically on assumptions about initial conditions (the time of core formation), the time-varying distribution of radioactive elements within the planet, thermal conductivities, the efficiency of convective heat transport, and perhaps most importantly, the amount of a lighter element such as sulfur that is mixed with the iron. Fricker *et al.* (1976) found that a liquid outer shell 500 km thick may persist to the present if one accounts for a thermal barrier at a core–mantle boundary (CMB) between liquid iron and solid silicate. The thermal barrier results from the higher melting point and lower thermal conductivity of the silicates. The liquid iron core temperature rises above the melting curve near the CMB, thereby decreasing the core temperature gradient. If the solid mantle is convecting, however, it must contain a density of heat sources comparable to that of the Earth's mantle-wide average if the core is to remain molten (Cassen *et al.*, 1976). Schubert *et al.* (1988) indicated that an outer fluid core with a radius of ~1800 km and a thickness of ~500 km can be maintained if a small percentage of a lighter element such as sulfur is mixed into the core to reduce the melting temperature. Harder and Schubert (2001) discussed the consequences of a wide range of sulfur content in Mercury's core.

Since theoretical predictions about the nature of Mercury's core must remain inconclusive, empirical determination of the core properties is needed to constrain and focus the theories. Whether or not the core is found to be molten, the result will have profound effects on inferences about Mercury's rotational history (Peale and Boss, 1977), on theories of planetary magnetic field generation, and on theories of the thermal history (Schubert *et al.*, 1988).

In the following section "The Experiment", we show how the determination of the four parameters, C_{20}, C_{22}, θ, and ϕ_0, where C_{20} and C_{22} are the second degree and order gravitational harmonic coefficients, θ is Mercury's obliquity, and ϕ_0 is the amplitude of the forced libration, should unambiguously determine the existence and extent of a molten core (Peale, 1976). (The amplitude of libration is the maximum deviation of the angular orientation of Mercury about its spin axis relative to its orientation for uniform rotation of $\psi = 1.5n$, where n is the orbital mean motion.) The crucial conditions that the liquid core not follow the 88-day physical libration, but that it does follow the 250 000-year precession of the spin axis, are made clear in this section. We show further that these conditions are easily satisfied if the coupling between the core and mantle is viscous in nature. Alternative core–mantle coupling mechanisms of pressure forces on irregularities on the CMB (Hide, 1989), gravitational coupling between an axially asymmetric mantle and an axially asymmetric solid inner core (Szeto and Xu, 1997), and magnetic coupling between the conducting liquid core and a conducting layer at the inner surface of the mantle (Buffett, 1992; Love and Bloxham, 1994; Buffett *et al.*, 2000) are shown in "The Hide Mechanism", "Non-Axisymmetric Inner Core", and "Magnetic Coupling", respectively, not to frustrate the first condition. An expression for the uncertainty in core character as a function of the uncertainty in the measurements of each of the four parameters, C_{20}, C_{22}, θ, and ϕ_0, is developed in "Restrictions on the Measurement Uncertainties". The capabilities of the nominal MESSENGER (MErcury Surface, Space ENvironment, Geochemistry, Ranging) mission to Mercury to measure these four parameters sufficiently accurately to establish unambiguously the nature of Mercury's core are pointed out in this section. We also describe briefly two proposed Earth-based radar experiments that promise to determine the obliquity and libration amplitude to precisions on the order of 1 arcsec. Coupled with a possible fractional uncertainty of 0.001 in the MESSENGER determination of the gravitational harmonic coefficients, such precision leads to an uncertainty in the ratio of the moment of inertia of Mercury's mantle to the total moment of inertia of a few percent.

THE EXPERIMENT

The upcoming MESSENGER orbiter mission to Mercury (Solomon *et al.*, 2001) with onboard instrumentation capable of measuring C_{20}, C_{22}, θ, and ϕ_0 (Zuber and Smith, 1997; Smith *et al.*, 2001), and the possibility of precision measurements of Mercury's spin geometry with radar interferometry techniques (Holin, 1999; Slade *et al.*, 1999) make a reexamination of this proposal to determine the nature of Mercury's core particularly relevant. To develop the method, we shall assume that the two necessary conditions on the core–mantle interaction for the experiment to work, (1) the core must *not* follow the 88-day physical libration of the mantle, and (2) the core must follow the mantle on the timescale of the 250 000-year precession of the spin in Cassini state 1 (see Peale, 1969, for a discussion of Cassini states), are satisfied. Then we will show that an extremely wide range of core viscosities, easily spanning all likely values, is consistent with the conditions.

The physical libration of the mantle about the mean resonant angular velocity arises from the periodically reversing torque on the permanent deformation as Mercury rotates relative to the Sun. The amplitude of this libration is given by (Peale, 1972)

$$\phi_0 = \frac{3}{2}\left(\frac{B-A}{C_m}\right)\left(1-11e^2+\frac{959}{48}e^4+\cdots\right) \tag{1}$$

where $A < B < C$ are the principal moments of inertia of the entire planet, and e is the orbital eccentricity. The moment of inertia C_m in the denominator of Eq. (1) is that of the mantle (plus crust) alone, since the core does not follow the libration. The full core, and for the time being the solid inner core, are assumed axially symmetric so they do not contribute to $B - A$. Dissipative processes will carry Mercury to rotational Cassini state 1 (where spin, orbit precessional, and orbital angular velocities remain coplanar) with an obliquity θ close to 0° (Peale, 1988). This condition leads to a constraint,

$$K_1(\theta)\left(\frac{C-A}{C}\right)+K_2(\theta)\left(\frac{B-A}{C}\right)=K_3(\theta) \tag{2}$$

where K_1, K_2, and K_3 are known functions whose forms are given in Peale (1969), and the moment of inertia in the denominator is now that of the entire planet since the core is assumed to follow the precession. Note that the precession here is not the relatively rapid precession of the spin about the Cassini state in the frame rotating with the orbit, which the core is not likely to follow, but it is the precession of the orbit (with the much longer period) in which frame the spin vector is locked if Mercury occupies the exact Cassini state.

We have made the assumption that Mercury occupies Cassini state 1. But the obliquity corresponding to this state depends on the orbital inclination to the Laplacian plane ι and on the orbital eccentricity e, both of which are variable (Table 1). If the precession period of the angular momentum around Cassini state 1 is much less than the 10^5 to 10^6 year periods for the variations in the orbital parameters, the solid angle traced

TABLE 1. Variations in Mercury orbit parameters (Cohen *et al.*, 1973).

Variations	Timescale
$0.11 \le e \le 0.24$	10^6 year period
0.006 amplitude variation	10^5 year period
$5° \le \iota \le 10°$	10^6 year period
0.25° amplitude variation	10^5 year period

out by the spin angular momentum is an adiabatic invariant (Goldreich and Toomre, 1969). This precession period is ~1800 years for the Anderson *et al.* (1987) values of $(C-A/2-B/2)/C \simeq (B-A)/C \simeq 10^{-4}$, and therefore we expect Mercury to be very close to the instantaneous location of the Cassini state (Peale, 1974).

The second degree and order gravitational harmonic coefficients (unnormalized) are expressed in terms of the moment differences as follows:

$$\begin{aligned} C_{20} &= -\frac{C-A}{MR^2}+\frac{1}{2}\frac{B-A}{MR^2} = -(6.0 \pm 2.0) \times 10^{-5}, \\ C_{22} &= \frac{B-A}{4MR^2} = (1.0 \pm 0.5) \times 10^{-5} \end{aligned} \tag{3}$$

where M and R are Mercury's mass and radius, respectively, and where the numerical values are estimated for Mercury from *Mariner 10* flyby data (Anderson *et al.*, 1987). It is relatively easy to define a coordinate system on Mercury that is sufficiently close to the principal axis system that the harmonic coefficient S_{22} and coefficients of degree 2 and order 1 are negligibly small. The axis of minimum moment of inertia deviates from being aligned toward the Sun at each perihelion passage by no more than the very small obliquity, because Mercury's occupancy of the 3:2 spin orbit resonance means this axis tries to be aligned with the Sun at perihelion and any librations in longitude about this state should be completely damped (Colombo, 1965; Goldreich and Peale, 1966). The spin axis must be essentially coincident with the axis of maximum moment of inertia because internal dissipation will damp any wobble (*e.g.*, Peale, 1973). Equation (3) can be solved for $(C-A)/MR^2$ and $(B-A)/MR^2$ in terms of C_{20} and C_{22}, determined by tracking the orbiting spacecraft. Substitution of the solutions of Eq. (3) into Eq. (2) yields a numerical value for C/MR^2, since the K_i are known once the obliquity θ is measured.

Measurement of the amplitude ϕ_0 of the physical libration determines $(B-A)/C_m$ (Eq. (1)), from which three known factors give

$$\left(\frac{C_m}{B-A}\right)\left(\frac{B-A}{MR^2}\right)\left(\frac{MR^2}{C}\right) = \frac{C_m}{C} \le 1 \tag{4}$$

A value of C_m/C of 1 would indicate a core firmly coupled to the mantle and most likely solid. If the entire core or the outer part is fluid, $C_m/C \simeq 0.5$ for the large core radius ($R_c \simeq 0.75R$) in current models of the interior (Cassen *et al.*, 1976).

We relax the assumption that the inner core is axially symmetric in the section "Non-Axisymmetric Inner Core", where we show that the gravitational torque between a triaxial inner core and a triaxial mantle does not affect the amplitude of forced libration of the mantle. Hence, our assumption that $B-A = B_m - A_m$ derived from this amplitude in the first factor in Eq. (4) remains intact. However, a triaxial core would contribute to the $B-A$ derived from C_{22} in the second factor of Eq. (4), and the cancellation would no longer be exact. To estimate how much $B-A$ in the second factor could differ from $B_m - A_m$, we model the inner core as a homogeneous prolate ellipsoid with mean density of the largest inner core in Table 2 (8.4 Mg/m^3) and with principal axes aligned with those of the mantle. This inner core is embedded in a fluid outer core whose density at the inner core boundary (ICB) is 7.8 Mg/m^3, where the densities follow from the model of Siegfried and Solomon (1974) and where we assume that the CMB is axially symmetric. We can think of the solid inner core as the sum of an ellipsoid with the same density as the fluid outer core at the ICB plus a superposed ellipsoid of density 0.6 Mg/m^3. Only the latter contributes to $B-A$. If $a_{ic} > b_{ic} > c_{ic}$ are the semi-principal axes of the inner core ellipsoid, and $a_m > b_m > c_m$ are the same for the mantle, the total $B-A$ as the sum of the mantle plus inner core is

$$B-A = \left(\frac{3.3}{5.4}\right)\frac{1}{5}M\left(a_m^2-b_m^2\right)+\frac{1}{5}M_{ic}\left(a_{ic}^2-b_{ic}^2\right)\left(1-\frac{\rho_f}{\langle\rho_{ic}\rangle}\right) \tag{5}$$

where ρ_f is the fluid density at the ICB and the angular brackets denote the average value of the inner core density. In Eq. (5) the expressions for the principal moments of inertia of a homogeneous ellipsoid are used, where the subtracted inner cavity in the mantle for both A_m and B_m cancels in their difference. The first fraction on the right-hand side of Eq. (5) is the ratio of the mantle density to the average density of the entire planet. If we assume that $(a_{ic}, b_{ic}) = R_{ic}\,(a_m, b_m)/R$, the contribution to $B-A$ from the inner core is ~3% of that from the mantle with the parameters of Table 2. The same procedure applied to the inner cores of radii 1500 and 750 km from Table 2 yields contributions to $B-A$ from the inner core of <1% and <0.001% of that from the mantle, respectively. The increase in ρ_{ic} with depth would tend to reduce these percentages still further. Although we consider the effect of a triaxial core here and in the section "Non-Axisymmetric Inner Core", it is not clear that the solid core can sustain a significant deviation from axial symmetry. At normal pressures, iron near its melting point loses most of its ability to oppose sustained shear stress, and one can speculate that at the high pressures of planetary interiors a similar weakness prevails. Still, we shall consider how the possible difference between $B-A$ determined from C_{22} and $B_m - A_m$ affects the uncertainty in C_m/C in the section "Restrictions on the Measurement Uncertainties" below.

TABLE 2. Mercury parameters.

	Planet	Mantle	Core	$IC_{S/Fe}$ = 0.002*	S/Fe = 0.01	S/Fe = 0.05
Radius (km)	2440	2440	1840	1700	1500	750
Mass (kg)	3.3×10^{23}	1.15×10^{23}	2.15×10^{23}	1.73×10^{23}	1.21×10^{23}	1.61×10^{22}
Density† (Mg/m^3)	5.42	3.3	7.6–9.3	7.8–9.3	8.2–9.3	9.1–9.3
C‡ ($kg\ m^2$)	6.46×10^{35}	3.62×10^{35}	2.84×10^{35}	1.96×10^{35}	1.08×10^{35}	3.62×10^{33}

*The columns labeled with values of S/Fe indicate the solid inner core (IC) properties corresponding to that mass fraction of sulfur in the complete iron core (Schubert *et al.*, 1988).
†The interior density distribution is from Siegfried and Solomon (1974).
‡C is the moment of inertia.

If viscosity of the molten core material is the primary means of coupling the core motion to that of the mantle, two time constants for the relaxation of relative motion between a molten core and solid mantle motion are given by (Peale, 1988):

$$\tau_1 = \frac{R_c}{(\nu\dot{\psi})^{1/2}}, \quad \tau_2 = \frac{R_c^2}{\nu} \tag{6}$$

where the first applies to small viscosities and the latter to large viscosities, with $\dot{\psi}$ being Mercury's spin rate and ν the kinematic viscosity. If $\tau_i \gg 88$ days, the core will not follow the mantle during the latter's 88-day libration, and if $\tau_i \ll 250\,000$ years, the core will follow the precession of the mantle angular momentum as it follows the orbit precession in Cassini state 1. These conditions correspond to

$$4 \times 10^{-8} < \nu < 5 \times 10^4 \text{ to } 4 \times 10^5 \text{ m}^2/\text{s} \tag{7}$$

As this range includes all possible values for the viscosity of likely core material (*e.g.*, Gans, 1972; de Wijs *et al.*, 1998), the experiment should work unless some other type of coupling can force the core to follow the 88-day libration of the mantle. That this is not the case is demonstrated in the next three sections.

THE HIDE MECHANISM

Hide (1989) has suggested that pressure forces on irregularities on the Earth's CMB can lead to the observed fluctuations in the length-of-day (LOD) with amplitudes as large as 5×10^{-3} s and timescales on the order of decades. For the purposes of evaluating a representative torque on the mantle due to such forces we assume that the variation in the LOD is simply periodic:

$$P = P_0 + 5 \times 10^{-3}\text{ (s)} \cos \omega t \tag{8}$$

where P_0 = 86 164 s is the sidereal period of Earth's rotation and $\omega = 2\pi/10$ years. Choosing the minimal 10-year period will maximize the necessary torque to reach $\Delta P = 5 \times 10^{-3}$ s. With $\dot{\psi} = 2\pi/P$,

$$\frac{dP}{dt} = -\frac{2\pi}{\dot{\psi}^2}\frac{d\dot{\psi}}{dt} = 9.95 \times 10^{-11} \sin \omega t \tag{9}$$

$$C_{m\oplus}\frac{d\dot{\psi}}{dt} = -8.42 \times 10^{-20} C_{m\oplus} \sin \omega t \tag{10}$$

where Eq. (10) gives the periodic torque on the mantle due to the pressure forces on the CMB and $C_{m\oplus}$ is the moment of inertia of the Earth's mantle (plus crust) about the rotation axis.

From Stacey (1992), the moment of inertia of the entire Earth about the spin axis is $C_\oplus = 0.3314\, M_\oplus R_\oplus^2 = 8.0363 \times 10^{37}$ kg m^2, where $M_\oplus = 5.9737 \times 10^{24}$ kg, and $R_\oplus = 6371.03$ km. From the distribution of core mass given in the preliminary reference Earth model (Dziewonski and Anderson, 1981), the moment of inertia of the core $C_{c\oplus} = 0.39\, M_{c\oplus} R_{c\oplus}^2 = 0.92 \times 10^{37}$ kg m^2, with core mass $M_{c\oplus} = 1.95 \times 10^{24}$ kg and core radius $R_{c\oplus} = 3480$ km (Stacey, 1992). So the moment of inertia of Earth's mantle is

$$C_{m\oplus} = C_\oplus - C_{c\oplus} = 7.12 \times 10^{37} \text{ kg m}^2 \tag{11}$$

The maximum torque on the Earth's mantle leading to the 5×10^{-3} s decade fluctuations in the LOD is (Eqs. (10) and (11)), then 6.00×10^{18} N m.

To scale this torque to Mercury, we note that dynamic pressure $\sigma_\oplus \approx \rho u_\oplus^2$, where ρ is the density of the fluid at the CMB and $u_\oplus \approx 0.0003$ m/s (Hide, 1989) is the assumed relative velocity of the fluid with respect to the mantle. (One estimate of this relative fluid velocity comes from the drift of the non-dipole component of the Earth's magnetic field of ~0.2°/ year (Lambeck, 1980).) Then

$$\frac{T}{T_\oplus} = \frac{R_c}{R_{c\oplus}}\frac{\sigma}{\sigma_\oplus}\frac{A_c}{A_{c\oplus}} = \frac{\rho_{fc}}{\rho_{fc\oplus}}\frac{u^2}{u_\oplus^2}\frac{R_c^3}{R_{c\oplus}^3} \tag{12}$$

where the left-hand side is the ratio of the torques on the respective mantles, R_c = 1840 km and $R_{c\oplus}$ = 3480 km are the respective core radii, A_c and $A_{c\oplus}$ are the areas of the inner surfaces of the mantles, u is a representative relative velocity between Mercury's mantle and the adjacent fluid core, and

$\rho_c/\rho_{fc\oplus} \approx 7.6/9.3$ (Siegfried and Solomon, 1974; Dziewonski and Anderson, 1981) is the ratio of densities of fluid core material at the CMBs. This scaling assumes the topography of Mercury's CMB is similar to Hide's assumptions about the Earth's CMB topography that is necessary to cause the millisecond variations in the LOD.

We estimate a maximum relative velocity between the core fluid and mantle by assuming the core to be uniformly rotating during the libration of the mantle about this rotation rate. From Eqs. (1) and (3), we have

$$\phi_0 = 3.42 C_{22} \frac{MR^2}{C} \frac{C}{C_m} \tag{13}$$

where $C_{22} = (1.0 \pm 0.5) \times 10^{-5}$ (Anderson *et al.*, 1987).

Properties of Mercury's interior shown in Table 2 follow from Schubert *et al.* (1988) with the density distribution of Siegfried and Solomon (1974) cited therein. As before, the subscripts "c", "m", and "ic" designate core, mantle, and inner core, respectively, and those symbols without subscripts refer to the entire planet. For the model of Table 2, $C_m/C = 0.56$ and $C/MR^2 = 0.33$ such that $19 \lesssim \phi_0 \lesssim 57$ arcsec. If we choose $\phi_0 = 40$ arcsec,

$$\begin{aligned} \phi &= 40(\text{arcsec}) \sin nt, \\ \frac{d\phi}{dt} &= 1.61 \times 10^{-10} \cos nt \ (\text{rad/s}) \end{aligned} \tag{14}$$

where $n = 2\pi/(88 \text{ days})$ is the orbital mean motion, and the zero of t is chosen for proper phasing. From Fig. 2 below we see that the first part of Eq. (14) is a reasonably good approximation, but that the second part is not, although the coefficient provides a measure of the maximum deviation in angular velocity from the mean value.

The maximum relative velocity between the core liquid and the mantle with the core fixed in the rotating frame is thus $u = 1.61 \times 10^{-10} R_c = 0.0003$ m/s, essentially identical to the value estimated for the Earth. Substitution of this value of u into Eq. (12) yields

$$T = 0.113 T_\oplus = 6.8 \times 10^{17} \text{ N m} \tag{15}$$

If we assume the maximum torque is sustained for all relative velocities and abandon uniform core rotation, then the relative angular velocity of the mantle and core, with the latter considered a rigid body coupled to the mantle only through the mutual Hide torque, varies as

$$\frac{d}{dt}(\dot\psi_m - \dot\psi_c) = -T\left(\frac{1}{C_m} + \frac{1}{C_c}\right) \text{sign}(\dot\psi_m - \dot\psi_c) \tag{16}$$

such that

$$\dot\psi_m - \dot\psi_c = \left(\dot\psi_m^0 - \dot\psi_c^0\right) - T\left(\frac{1}{C_m} + \frac{1}{C_c}\right) t \tag{17}$$

where the superscript 0 designates initial value with $\dot\psi_m^0 - \dot\psi_c^0 > 0$. If the initial relative velocity is the maximum attained during the libration (core rotating at the mean spin angular velocity) of 1.61×10^{-10} rad/s, and parameter values from Table 2 are used, the core and mantle become synchronously rotating after 1.2 years. This time is already long compared with the 44 days between reversals of the forced libration, and the actual timescale will be much longer since the maximum torque will not prevail continuously. In fact, if the core is held to a uniform rotation during the libration, and the Hide torque from Eq. (12) is added to the equation of libration (Goldreich and Peale, 1966) of the mantle as follows:

$$\begin{aligned} \ddot\gamma + \frac{3}{2}\frac{B-A}{C_m}\frac{GM_\odot}{a^3}\frac{(1+e\cos f)^3}{(1-e^2)^3}\sin(3nt + 2\gamma - 2f) \\ = -\frac{T_\oplus}{C_m}\frac{\rho_{fc}}{\rho_{fc\oplus}}\frac{R_c^5\dot\gamma^2}{R_{c\oplus}^3 u_\oplus^2}\text{sign}(\dot\gamma) \\ = -62\dot\gamma^2 \text{ sign}(\dot\gamma) \end{aligned} \tag{18}$$

then the change in the forced libration is essentially undetectable. In Eq. (18), n is the orbital mean motion of Mercury, $\dot\gamma = \dot\psi_m - 1.5n$ is the deviation of the rotational angular velocity from the resonant value, a is the orbital semimajor axis, $M_\odot$ is the solar mass, and f is the true anomaly. Since we assume the core is uniformly rotating to realize the maximum torque, $\dot\psi_c = 1.5n$ and $\dot\phi = \dot\gamma$ in the expression $u = R_c\dot\phi$. It could be the case that the combination of CMB topography and relative fluid velocity u on Mercury is sufficiently different from that of the Earth that the timescale for topographic coupling is reduced from our estimate. Although there is no reason to believe this is the case and topographic coupling may even be much weaker than we have supposed, the possibility that it could be larger should be kept in mind when interpreting the MESSENGER data.

With this caution, we conclude that an irregular CMB will not force Mercury's core to follow the mantle on the 88-day forced libration timescale, if the effect of the irregularities is scaled from the Earth. At the same time, this coupling is much stronger than the viscous coupling and makes the condition that the core follow the mantle on the 250 000-year timescale more secure—if, in fact, the CMB is not axisymmetric. (Up to 3 km of topography at the Earth's CMB and a core ellipticity of 0.2 km have been found from recent analysis of body wave travel times (Sze and van der Hilst, 2002).)

NON-AXISYMMETRIC INNER CORE

Because the core of Mercury is so large relative to the planetary radius, and because for low sulfur content the solid inner core would also occupy a large fraction of the core volume

(Table 2; Schubert *et al.*, 1988), a deviation from axial symmetry for the solid inner core might lead to a significant gravitational coupling between the inner core and the mantle (Szeto and Xu, 1997). Can such a coupling drag a large inner core along with the forced libration of the mantle and mask the librational signature of a liquid outer core? We show that although the inner core might affect the free libration of the mantle, the 88-day forced libration of the mantle is superposed on any larger librations with essentially undiminished amplitude.

According to Szeto and Xu (1997), the gravitational torque on the solid inner core from a triaxially deformed mantle is

$$T_{\mathrm{ic}} = \frac{4\pi}{5}\alpha G\rho_{\mathrm{m}}(\kappa_2 - \kappa_1)(B_{\mathrm{ic}} - A_{\mathrm{ic}})\sin 2(\psi_{\mathrm{m}} - \psi_{\mathrm{ic}}) \quad (19)$$

where $\kappa_i = (a_{\mathrm{m}i} - b_{\mathrm{m}i})/a_{\mathrm{m}i}$, $a_{\mathrm{m}i} > b_{\mathrm{m}i}$, $i = 1,2$, are the lengths of the principal ellipsoid axes in the equatorial plane for the inner and outer surfaces of the mantle, respectively, $B_{\mathrm{ic}} > A_{\mathrm{ic}}$ are the principal moments of inertia of the inner core, ρ_{m} is the density of the mantle material, and $\alpha = 1 - \rho_{\mathrm{f}}/\langle\rho_{\mathrm{ic}}\rangle$ is a measure of the compensation of the gravitational torque on the inner core by the gravitationally induced pressure forces from the fluid outer core. Another way to look at this last factor is to realize that the gravitational effect on an asymmetric, homogeneous inner core must disappear if it is embedded in a homogeneous fluid of equal density. Here ρ_{f} is the liquid core density at the ICB and the angled brackets indicate the average of the inner core density as before.

To maximize the torque, we assume that the inner surface of the mantle (*i.e.*, the CMB) is axially symmetric, so that $\kappa_1 = 0$. To find κ_2, we assume the measured value of $C_{22} = 10^{-5}$ is due to a surface mass distribution of the form

$$\sigma_{\mathrm{s}} = \rho_{\mathrm{m}}\Delta r_0 P_{22}(\cos\theta')\cos 2\phi' \quad (20)$$

where (θ', ϕ') are spherical polar coordinates, $P_{22} = 3\sin^2\theta'$ is the associated Legendre function of degree and order 2, and Δr_0 is a radial increment scaling the thickness of the harmonic surface layer. Then

$$\begin{aligned} B - A &= \int_0^{\pi}\int_0^{2\pi}(x^2 - y^2)\sigma_{\mathrm{s}}R^2\sin\theta'\,d\theta'\,d\phi' \\ &= \frac{16\pi}{5}R^4\rho_{\mathrm{m}}\Delta r_0 \\ &= 4\times 10^{-5}MR^2 \end{aligned} \quad (21)$$

where the last form follows from the definition and value of C_{22}. Then

$$\Delta r_0 \approx 0.07\ \mathrm{km} \quad (22)$$

In Eq. (21), x and y are the cartesian coordinates of a point on the surface relative to an origin at the center of the planet, and the integrand is just the square of a spherical harmonic whose integral is well known. From Eq. (20) we can deduce $\kappa_2 = 1.74\times 10^{-4}$, and Eq. (19) can be written

$$\begin{aligned} T_{\mathrm{ic}} &= 1.86\times 10^{24}\frac{B_{\mathrm{ic}} - A_{\mathrm{ic}}}{C_{\mathrm{ic}}}\sin 2(\psi_{\mathrm{m}} - \psi_{\mathrm{ic}}) \\ &= 2.23\times 10^{20}\sin(\psi_{\mathrm{m}} - \psi_{\mathrm{ic}})\ \mathrm{N\,m} \end{aligned} \quad (23)$$

where we have chosen the maximum inner core radius of 1700 km from Table 2 to maximize the gravitational interaction, $\alpha = 0.1$ is assumed (Szeto and Xu, 1997), and we have assumed $C_{\mathrm{ic}}/M_{\mathrm{ic}}R_{\mathrm{ic}}^2 = 0.39$ from Table 2 and $(B_{\mathrm{ic}} - A_{\mathrm{ic}})/C_{\mathrm{ic}} = 1.22\times 10^{-4}$.

To determine the effect of the axially asymmetric inner core on the librations of the mantle, we write two coupled equations for the libration of the mantle and inner core, respectively:

$$\begin{aligned} C_{\mathrm{m}}\ddot{\psi}_{\mathrm{m}} + \frac{3}{2}(B_{\mathrm{m}} - A_{\mathrm{m}})\frac{GM_\odot}{r^3}\sin 2\delta_{\mathrm{m}} &= -K\sin 2(\psi_{\mathrm{m}} - \psi_{\mathrm{ic}}) \\ C_{\mathrm{ic}}\ddot{\psi}_{\mathrm{ic}} + \alpha\frac{3}{2}(B_{\mathrm{ic}} - A_{\mathrm{ic}})\frac{GM_\odot}{r^3}\sin 2\delta_{\mathrm{ic}} &= K\sin 2(\psi_{\mathrm{m}} - \psi_{\mathrm{ic}}) \end{aligned} \quad (24)$$

where K is the numerical coefficient given in Eq. (23), r is the Sun–Mercury distance, and $\delta_{\mathrm{m,ic}}$ are the angles between the Mercury–Sun line and the long axes of the mantle and inner core, respectively. Note that α appears in the second of Eq. (24) to account for the effect of the liquid outer core. From Goldreich and Peale (1966) $\delta_{\mathrm{m,ic}} = \psi_{\mathrm{m,ic}} - f$, where f is the true anomaly of the orbital motion. We write $\psi_{\mathrm{m,ic}} = 1.5\mathcal{M} + \gamma_{\mathrm{m,ic}}$ with $\mathcal{M} = n(t - t_0)$ being the mean anomaly of the orbital motion such that $\gamma_{\mathrm{m,ic}} = \delta_{\mathrm{m,ic}}$ when Mercury is at perihelion. The latter follows from the fact that Mercury's mean spin rate is $1.5n$. If we substitute these definitions into Eqs. (24), replace r by its definition in terms of a, e, and f, divide through by $GM_\odot/a^3 = n^2$, and change the independent variable t to the dimensionless nt so that an orbital period equals 2π, we arrive at the following set of differential equations that can be solved numerically:

$$\begin{aligned} \ddot{\gamma}_{\mathrm{m}} &= -\frac{3}{2}\left(\frac{B_{\mathrm{m}} - A_{\mathrm{m}}}{C_{\mathrm{m}}}\right)\frac{(1 + e\cos f)^3}{(1 - e^2)^3}\sin(3t + 2\gamma_{\mathrm{m}} - 2f) - \\ &\quad \frac{Ka^3}{GM_\odot C_{\mathrm{m}}}\sin 2(\gamma_{\mathrm{m}} - \gamma_{\mathrm{ic}}) - \xi(\dot{\gamma}_{\mathrm{m}} - \dot{\gamma}_{\mathrm{ic}}), \\ \ddot{\gamma}_{\mathrm{ic}} &= -\frac{3}{2}\alpha\left(\frac{B_{\mathrm{ic}} - A_{\mathrm{ic}}}{C_{\mathrm{ic}}}\right)\frac{(1 + e\cos f)^3}{(1 - e^2)^3}\sin(3t + 2\gamma_{\mathrm{ic}} - 2f) + \\ &\quad \frac{Ka^3}{GM_\odot C_{\mathrm{ic}}}\sin 2(\gamma_{\mathrm{m}} - \gamma_{\mathrm{ic}}) + \xi(\dot{\gamma}_{\mathrm{m}} - \dot{\gamma}_{\mathrm{ic}}), \\ \dot{f} &= \sqrt{\frac{1}{(1 - e^2)^3}}(1 + e\cos f)^2 \end{aligned} \quad (25)$$

We have arbitrarily added a damping term in Eqs. (25), where ξ is a coefficient of arbitrary magnitude.

First we determine the librations of the mantle for an axisymmetric core, which is accomplished in Eqs. (25) by setting $\alpha = K = 0$. Figure 1 shows forced libration ϕ with an amplitude of ~40 arcsec. Initial conditions are chosen in general with $\phi = \gamma_m = \gamma_{ic} = f = 0$ and values of $\dot{\gamma}_m = \dot{\phi}$ and $\dot{\gamma}_{ic}$ chosen to determine the free librations. The conditions on the inner core are irrelevant when $\alpha = 0$. The free libration is expected to be completely damped, but we have been surprised in the past and a small free libration with a period of ~49.5 Mercury orbit periods is illustrated in Fig. 1. It is unlikely we will be able to distinguish a free libration within the few orbital periods available during the MESSENGER mission. However, the combination of MESSENGER and the European Space Agency (ESA) mission, BepiColombo (Anselm and Scoon, 2001), which will arrive at Mercury a few years later than MESSENGER, may allow a free libration to be detected if present.

Figure 2 shows the deviations of the physical libration angle, ϕ, from pure harmonic motion along with the librational angular velocity for the axially symmetric core case. The flat region of the velocity curve near perihelion results from the reversal in the gravitational torque while the orbital angular velocity exceeds the rotational angular velocity. The Sun reverses its motion in Mercury's sky for a short time near perihelion. The value of $d\phi/dt$ as a function of orbital phase will prove useful in the radar determination of the libration amplitude described in the last section.

Figure 3 shows an expanded view of the forced libration for three Mercury periods with and without a molten outer core.

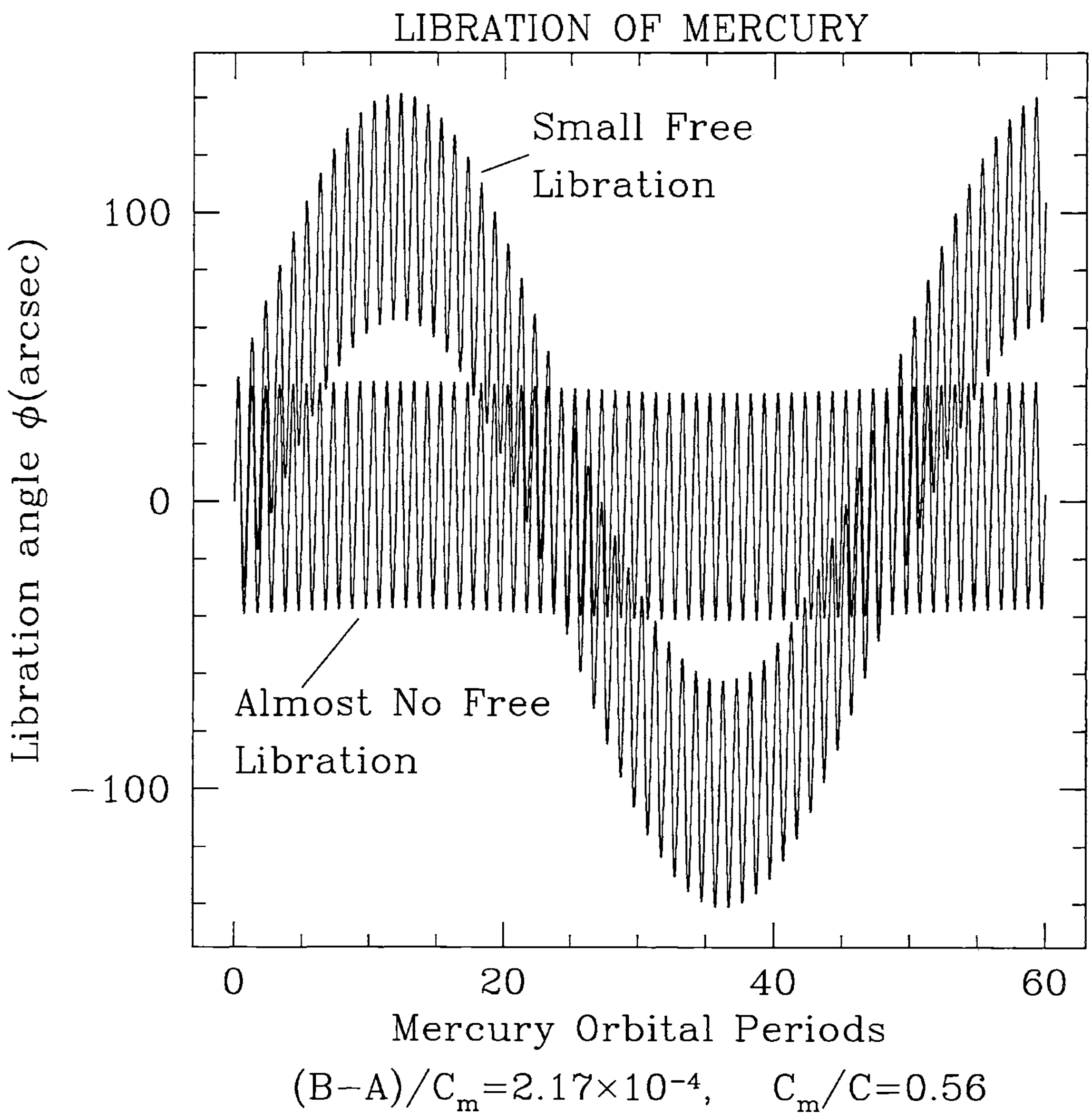

FIG. 1. Forced libration of Mercury for an axisymmetric inner core showing examples with almost no free libration and with a small free libration in addition to the forced libration with an 88-day period. The two curves correspond to different initial values of $\dot{\phi}$.

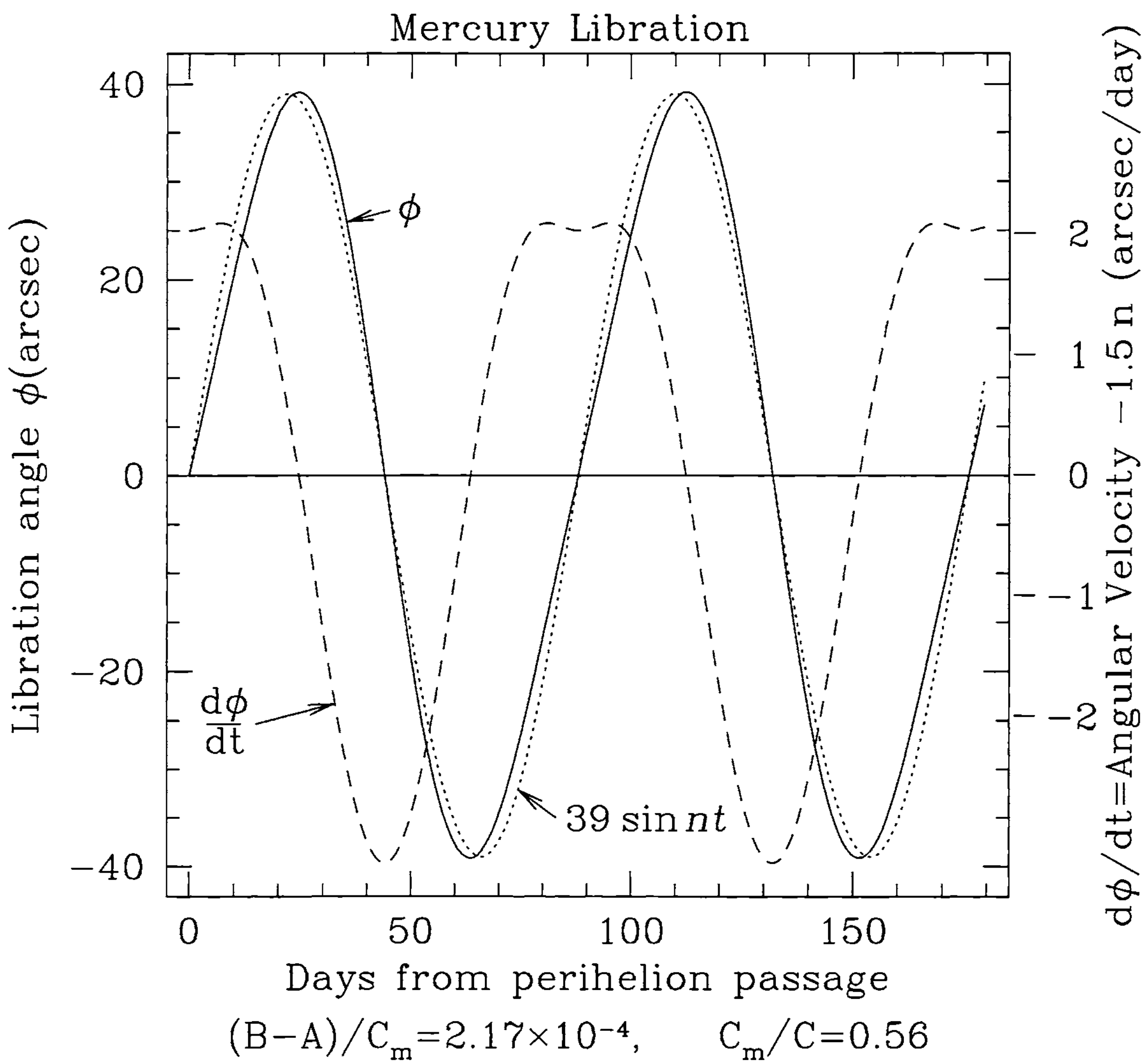

FIG. 2. Forced libration of Mercury for an axisymmetric inner core showing the deviations of the motion from the pure harmonic forms of Eqs. (14). The flat portion of $d\phi/dt$ results from the reversal of the gravitational torque on the asymmetric mantle near perihelion.

For the particular set of parameters chosen here, the amplitude in the former case is almost double that in the latter and forms the basis for determining the state of the core.

We can look at the librations of the inner core without the mantle by setting $(B - A)/C_{\rm m} = K = 0$. Our choice of $(B_{\rm ic} - A_{\rm ic})/C_{\rm ic} = 1.22 \times 10^{-4}$, the value for the entire planet determined from the *Mariner 10* flybys, yields a free libration period of ~206 orbit periods. In Fig. 4 we show the librations of both inner core and mantle with initial conditions corresponding to those that yielded no free libration when only the solar torque was involved. The forced 88-day libration is now superposed on a rather complex free libration. The core more or less follows the long period part of the free libration of the mantle, whose approximate period has increased to ~65 orbit periods. The frequency corresponding to this period is approximately the difference between the free libration frequencies of the mantle and inner core isolated in the solar field:

$$\frac{1}{49.5\text{ orbits}} - \frac{1}{206\text{ orbits}} = \frac{1}{65.2\text{ orbits}} \qquad (26)$$

A frequency equal to the sum of the two free libration frequencies also seems to be apparent in Fig. 4. If we consider the motion of the core and mantle to be governed solely by their gravitational interaction (as if the Sun were removed), the small oscillation period of the mutual alignment of the long axes is ~13.8 orbit periods. This is reasonably short compared with the free libration period, allowing the core and mantle to track each other on that timescale.

Figure 5 shows the consequences of choosing an arbitrary value of the damping coefficient $\xi = 0.005$ in Eqs. (25). The complex libration patterns are rather quickly suppressed, the core and mantle track very closely, and the free libration becomes a smooth sinusoid, with the 88-day forced librations still superposed. The continued relative motion between the core and mantle gradually damps the amplitude of the free

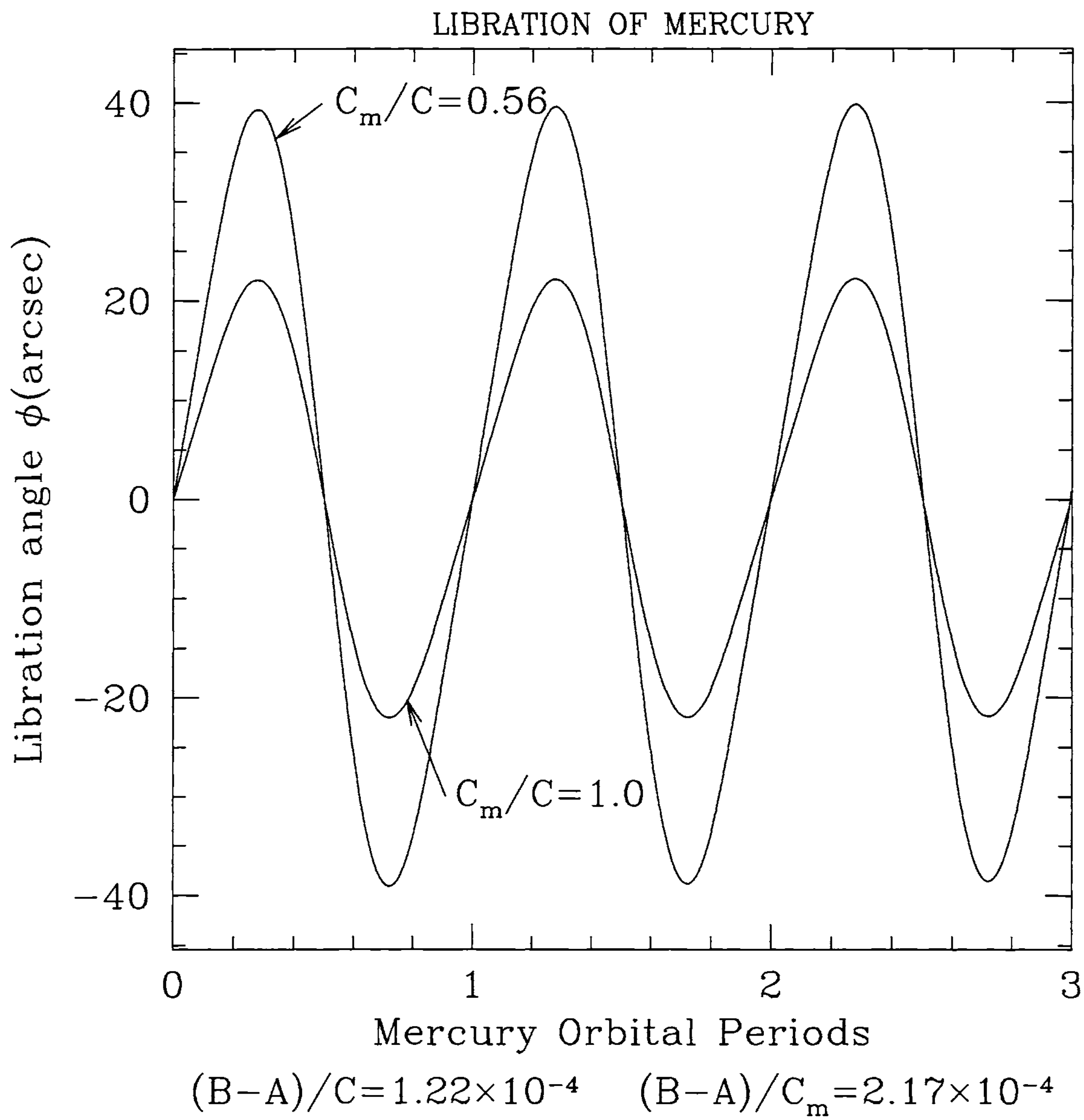

FIG. 3. Comparison of forced libration of Mercury for two cases, one where the outer core is liquid and one where the entire planet is solid.

libration, and the motion approaches the case of no free libration shown in Fig. 1.

The inner core with the moment differences we have assumed can have a profound effect on the free librations only if there has been a recent excitation of the free libration, and the damping has not yet caused the core to follow the mantle's free libration closely as in Fig. 5. Regardless of the state of free libration, the 88-day forced libration of the mantle is undiminished in amplitude. This is appreciated more if we reduce the initial angular velocity of the mantle from that used in Fig. 4 until the free libration virtually disappears. This is the state in which we expect to find Mercury. In this case the long axes in the equatorial planes of the core and mantle will be aligned, except for their respective forced libration. Since the angular separation $\psi_m - \psi_{ic}$ of the long axes can be at most the few tens of arc seconds of the physical libration, whereas $0 \le \sin 2\delta_{m,ic} \le 1$, Eqs. (24) shows that both core and mantle exhibit forced librations as if the other were axially symmetric; the equations are essentially decoupled. If we superpose the libration curve of the mantle in this case onto the libration curve of the mantle for an axially symmetric core shown in Fig. 3, the two curves are nearly indistinguishable. Hence, even with the largest of the Schubert *et al.* (1988) solid inner cores with $(B_{ic} - A_{ic})/C_{ic} = (B - A)/C$, the forced libration of the mantle is unaffected. At the same time, if there is a significant, recently excited free libration of the mantle for unknown reasons, the effects of the solid inner core, if deformed, may be detectable and its properties constrained, albeit only with the help of a later, second mission with measurements at least as precise as those from MESSENGER. Such capabilities are expected for

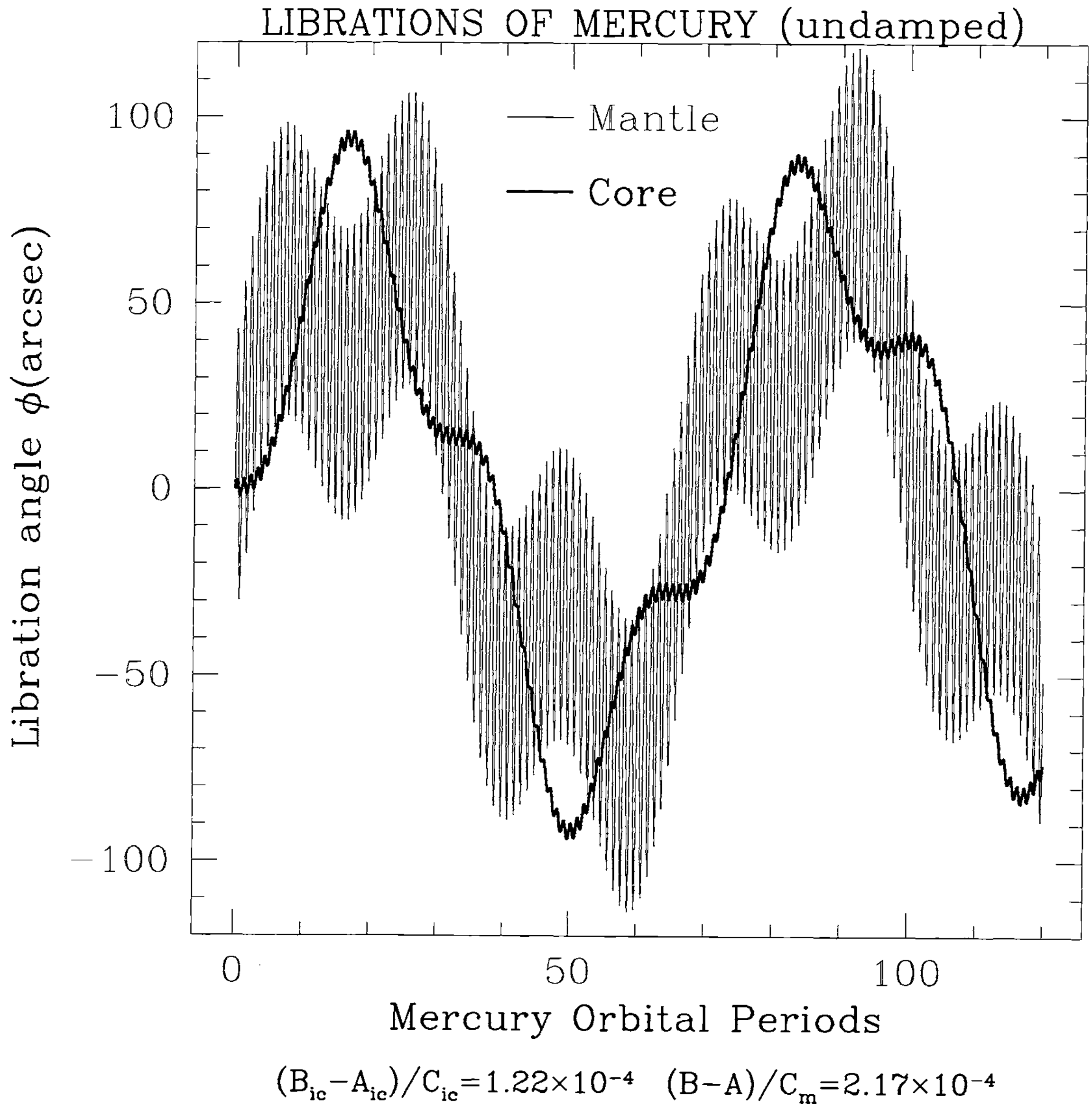

FIG. 4. Libration of Mercury's mantle and core coupled through gravitational interaction between axially asymmetric distributions of mass. The damping coefficient $\xi = 0$.

the BepiColombo mission mentioned above (Anselm and Scoon, 2001).

MAGNETIC COUPLING

The magnetic torque between the mantle and the liquid core is based on a model where field lines originating in the liquid core penetrate the mantle and are sheared due to relative motion between the mantle and core. A conducting layer in the mantle at the CMB and the conducting nature of the liquid core tend to anchor the field on both sides of the CMB, giving rise to a field distortion leading to the torque (Buffet, 1992). The conducting layer fixed to the mantle is hypothesized to result from liquid iron entrapped in silicate sediments at the CMB (Buffett *et al.*, 2000).

The torque between the liquid core and the mantle is then (Buffett, 1992; Eq. (52))

$$\overline{\Gamma}_m = \frac{1}{\mu_0}|\Phi| \int_S [B_0^r(r)]^2 [r^2 \overline{\omega} - (\mathbf{r} \cdot \overline{\omega})\mathbf{r}] dS \tag{27}$$

where r is here the radial coordinate reckoned from the center of the planet, Φ is a complex coupling constant that depends on the thickness of a rigid conducting layer in the mantle at the CMB, B_0^r is the radial component of the magnetic field at the CMB, the integral is taken over the spherical surface S at $r = R_c$, and $\overline{\omega}$ is the relative angular velocity between the core and mantle. In Eq. (27), we ignore the paramagnetism of the

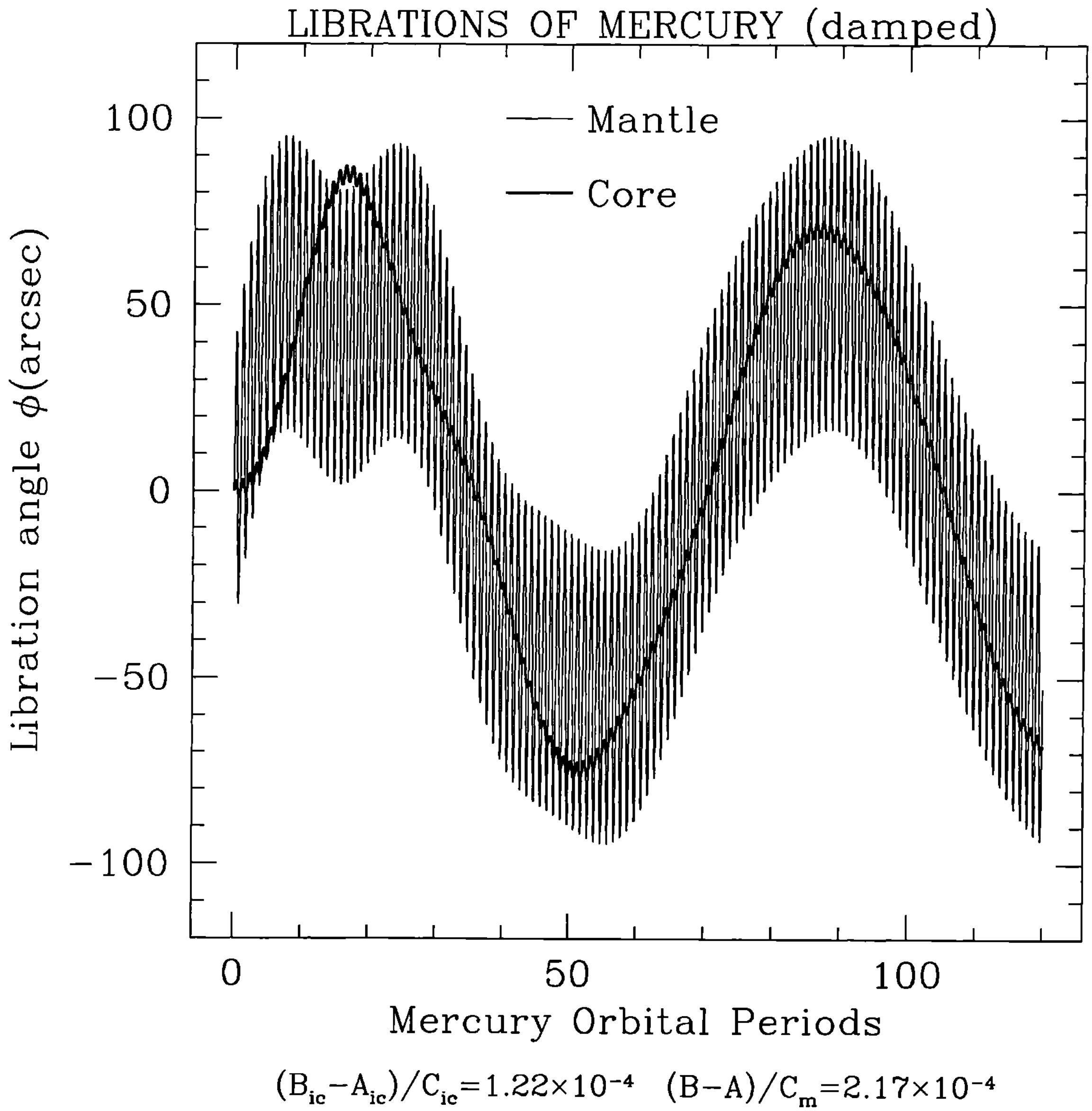

FIG. 5. Same as Fig. 4, except the damping coefficient $\xi = 0.005$. This shows how the complex librations and relative core–mantle motions are suppressed as the core follows the free librations of the mantle while both core and mantle exhibit their 88-day forced librations—each with undiminished amplitude.

hot iron by letting the permeability $\mu \rightarrow \mu_0$, the permeability of free space, and we ignore the phase shift between the perturbed magnetic field and the velocity field that causes the perturbation by using the absolute value of Φ. We shall investigate the decay of a step function perturbation to the relative angular velocity, where a phase shift is irrelevant. The value of Φ rises steeply with the thickness Δ of the conducting mantle layer at the CMB and reaches a nearly constant value of ~43 S/m when $\Delta > 200$ m for conductivity $\sigma = 5 \times 10^5$ S/m (Fig. 2; Buffett, 1992). For B_0^r we use the radial component of the simple dipole field whose axis is coincident with the spin axis:

$$B_0^r = \frac{\mu_0 m \cos\theta'}{2\pi r^3} \tag{28}$$

where m is the magnetic moment in A m^2 and θ' is the colatitude.

Substitution of Eq. (28) into Eq. (27), with $r = R_c$, $\overline{\omega} = \omega \mathbf{k}$ and $\mathbf{r} = r\hat{\mathbf{r}}$, where $\mathbf{k}$ and $\hat{\mathbf{r}}$ are unit vectors along the spin axis and radius vector, respectively, yields

$$\overline{\Gamma} = \frac{|\Phi|\mu_0 m^2 \omega}{2\pi R_c^2} \int_0^{\pi} \cos^2\theta' \sin^3\theta' \, d\theta' \mathbf{k} = \frac{2|\Phi|\mu_0 m^2 \omega(t)}{15\pi R_c^2} \mathbf{k} \tag{29}$$

where integration over the azimuthal angle ϕ' eliminates the components in the equatorial plane. If we assume that the core has infinite inertia and give the mantle a small angular velocity ω_0 relative to the core, the time constant for the decay of the relative angular velocity is

$$\tau = \frac{15\pi R_c^2 C_m}{2|\Phi| m^2 \mu_0} \quad (30)$$

where C_m is the moment of inertia of the mantle.

The value of the magnetic moment for Mercury is $(6 \pm 2) \times 10^{12}$ T m^3 $= \mu_0 m/(4\pi)$ if m is expressed in A m^2 (Russell *et al.*, 1988). We used $C_m = 3.62 \times 10^{35}$ kg m^2 and R_c = 1840 km from Table 2, and with $|\Phi|$ = 43 S/m and $\mu_0 = 4\pi \times 10^{-7}$ H/m, we find

$$\tau = 4.7 \times 10^6 \text{ years} \quad (31)$$

It is clear that, for this model, the magnetic torque will not affect the 88-day libration.

Alternatively for the magnetic coupling, we again invoke an analogy with the Earth. Electromagnetic coupling between a moving, fluid, conducting core and a conducting lower mantle has been proposed as a cause of the millisecond variations in the length of day. Love and Bloxham (1994) examined several mechanisms by which such a coupling could occur: (1) a poloidal couple resulting from the interaction of the poloidal field with currents induced by its time variation, (2) an advective couple that results from the dragging of poloidal field lines through a conducting mantle as we considered above, or (3) a leakage couple that results from the diffusion of toroidal magnetic field from the core's interior into the mantle. Love and Bloxham found that neither the poloidal couple nor the advective couple exhibit sufficient variability to account for the decadal LOD variations. Finally, a leakage couple sufficient to cause the LOD variations requires very strong and spatially complex toroidal fields, since there is significant cancellation when the magnetic stress is integrated over the CMB. According to Love and Bloxham, toroidal fields that could affect the LOD are too strong according to dynamo theory, produce electric fields at the surface that are stronger than those measured, and produce ohmic heating which either exceeds or contributes too large a fraction to the Earth's heat flow. Although Love and Bloxham qualified their conclusions by mentioning possible magnetic structures that would nullify their assumptions, it seems unlikely that the LOD fluctuations on the Earth could be caused by magnetic coupling.

Mercury's observed magnetic dipole moment of 6×10^{12} T m^3 (Russell *et al.*, 1988) compares with 8.1×10^{15} T m^3 for the Earth (Chapman and Bartels, 1940). If we assume that $B_0{}^r$ comes from the dipole component of the field, then the $R_c{}^2 C_m/m^2$ dependence in Eq. (30) means the ratio of time constants for the decay of a differential angular velocity $\tau \backslash \tau_\oplus \approx 2600$. Since we have already shown that torques sufficient to cause the LOD variations, when scaled to Mercury, do not significantly affect the 88-day libration amplitude of a mantle floating on a liquid core, the rejection of magnetic coupling for the LOD variations on the Earth means that the results of Love and Bloxham further strengthen our rejection of magnetic core–mantle coupling affecting the 88-day libration. Possibly the weakest part of the Love and Bloxham arguments concerns the torques associated with the toroidal magnetic field. This field geometry relies entirely on theoretical models of the dynamo and a large uncertainty in how the core heat flow contributes to the surface heat flow (G. Schubert, pers. comm., 2002), and the uncertainty is transferred to the torque estimates. On the other hand, the estimated magnetic torques fail by such a large factor that, even with these caveats, the magnetic torque is very unlikely to couple the core and mantle on the 44-day timescale for external torque reversal.

We conclude that the decoupling of Mercury's mantle from a liquid core on the 88-day timescale and the strong coupling of the mantle to the core on the 250 000-year timescale as deduced from considerations of viscous coupling are sustained in spite of possible topographical coupling, gravitational coupling, and magnetic coupling.

RESTRICTIONS ON THE MEASUREMENT UNCERTAINTIES

Measured ranges of values of C_{20} and C_{22} are given in Eqs. (3), and values of the obliquity and of the libration amplitude corresponding to the extreme values of these harmonics are

$$1.7 \lesssim \theta \lesssim 2.6 \text{ arcmin}, \qquad 20 \lesssim \phi_0 \lesssim 60 \text{ arcsec} \quad (32)$$

where θ follows from the solution of Eq. (2) and ϕ_0 from Eq. (1) with $\phi_0 = 0.854\ (B - A)/C_m$ for $e = 0.206$ and $C_m/C = 0.5$ and with $C/MR^2 = 0.35$ being assumed.

To estimate the required precision of the measurements for meaningful interpretation, we designate the four parameters whose nominal values are given in Eqs. (3) and (32) by η_i and write

$$\Delta\left(\frac{C_m}{C}\right) = \sum_i \frac{\partial}{\partial \eta_i}\left(\frac{C_m}{C}\right)\Delta\eta_i \quad (33)$$

which gives a maximum uncertainty of

$$\left.\frac{\Delta(C_m/C)}{(C_m/C)^0}\right|_{\max} = \sum_i \left| f_i \frac{\Delta\eta_i}{\eta_i^0} \right| \approx 22\% \quad (34)$$

The superscript 0 indicates the nominal values of the parameters, f_i = –0.83, 0.83, –1, and –1, respectively (Peale, 1981), for $\eta_i = C_{20}$, C_{22}, θ, and ϕ_0, and the numerical value corresponds to fractional uncertainties of 0.01, 0.01, 0.1, and

0.1, respectively, for the four parameters. The nominal values of the η_i are taken to be $C_{20}^0 = -6 \times 10^{-5}$, $C_{22}^0 = 1 \times 10^{-5}$, $\theta^0 = 3$ arcmin, and $\phi_0^0 = 30$ arcsec. If we assume an uncertainty in the value of the mantle contribution to C_{22} of 3% corresponding to the possible contribution of the maximum sized triaxial core in Table 2, the maximum uncertainty in C_m/C in Eq. (34) is only increased by a little over 2%. This is comparable to the low uncertainty expected in C_m/C obtained for the probably attainable lower measurement uncertainties for the parameters discussed below.

If we arbitrarily replace the second part of Eqs. (3) with $C_{22} = -C_{20}/8$ and let $C/MR^2 = 0.35$ as before, then $5.2 \gtrsim \theta \gtrsim 1.0$ arcmin for the expanded range $2 \times 10^{-5} \lesssim -C_{20} \lesssim 1 \times 10^{-4}$, with the corresponding range of ϕ_0 being shifted only slightly to smaller values. But because the angles are all small, the f_i are not significantly affected in the uncertainty estimate provided C_{22}/C_{20} remains approximately constant.

The maximum error from Eq. (34) would yield $C_m/C = 0.5 \pm 0.11$, which would distinguish an outer molten core from a completely solid core. Since the numerical coefficients in Eq. (34) are not very sensitive to the nominal values of the parameters, the error estimates remain the same for other reasonable values of the parameters if the *fractional* uncertainty of each parameter is unchanged. If, on the other hand, we again choose the fractional uncertainties in the η_i of 0.01, 0.01, 0.1, 0.1 as before, but let the nominal values of the four parameters go to the extremes shown in Eqs. (3) and (32), then from Eq. (34), $C_m/C = 0.5 \pm 0.24$ for all minimal nominal values, and $C_m/C = 0.5 \pm 0.065$ for all maximal nominal values. Even the worst case would resolve a molten outer core, although the core size would not be well constrained.

In a simulation of the MESSENGER mission scenario as of 1997 (Zuber and Smith, 1997), four Mercury years of data acquisition by the laser altimeter and the Deep Space Network (DSN) tracking systems were analyzed for the recovery of the gravity, topography, libration, and obliquity of Mercury. An x-band tracking system was assumed, and all tracking scenarios, solar conjunction periods, and other associated mission design parameters were included in the simulation. The tracking data, sampled at 10 s intervals, were assumed to be unbiased, characterized by 0.1 mm/s noise, and obtained by a single DSN tracking station located at Goldstone, California, with a 5° lower elevation limit. No data were acquired within 1° of the Sun, a time gap of nearly 2 days every 58 days. Except for the C_{20} and C_{22} terms, the *a priori* gravity model, an 8 × 8 spherical harmonic expansion, was generated from the model of normalized coefficients: $C_{\ell m}, S_{\ell m} \approx 8.0 \times 10^{-5}/\ell^2$. The harmonic coefficients are normalized by the factor

$$\sqrt{(\ell+m)!/[(\ell-m)!(2-\delta_{0m})(2\ell+1)]}$$

The normalized forms of C_{20} (~2.7×10^{-5}) and C_{22} (~1.6×10^{-5}) from Anderson *et al.* (1987) (Eq. (3)) were used for degree 2. The *a priori* topography model was a 16 × 16 degree and order spherical harmonic model of the lunar topography scaled to Mercury by the ratio of the inverse gravitational accelerations at the surfaces and with degree 1 terms set to zero.

The results of the analysis of the simulated altimetry and tracking data for gravity and libration parameters indicated recovery of the *a priori* degree 2 gravity coefficients to 0.5% and 0.2% for C_{20} and C_{22}, respectively, the amplitude of the libration to 25 m (~7% for $\phi_0 = 30$ arcsec), and the obliquity to 2 arcsec (~1% for $\theta = 3$ arcmin). Although the simulation was not an exact model of the MESSENGER mission as presently planned, it contained most of the orbital, dynamic, and operational constraints presently expected and suggests that MESSENGER will meet the conditions listed above for determining the existence of a fluid core.

Two ground-based radar experiments have been proposed to determine both θ and ϕ_0, possibly in advance of the determinations by MESSENGER. One such technique, called repeat orbit radar interferometry (RORI), relies on viewing a point on Mercury with the same geometry at two separate conjunctions, such that the respective subradar tracks cross (Slade *et al.*, 1999). The point on Mercury where the trajectories cross is identified by the appearance of fringes when the signals are superposed in a computer. This coincidence, together with the time interval between observations, constrains the right ascension and declination of the spin axis and the rotation rate. This technique suffers from the relatively few opportunities when the geometries of separate radar observations are nearly identical.

The other technique, called radar speckle displacement interferometry (RSDI), relies on determining the epoch and time delay that maximize the cross correlation of the speckle signal at two separate antennas separated by a long baseline (Holin, 1999). When a rough surface is illuminated by coherent radiation such as a laser or radar, the surface appears speckled because of the spatial variation of constructive and destructive interference. As Mercury rotates while illuminated with a continuous radar signal, this speckle pattern sweeps across the Earth while remaining frozen for distances much longer than an Earth diameter. The cross correlation of the speckle signal will be large with the proper time delay only if the antennas are aligned along the instantaneous velocity vector of the speckle pattern so both antennas see the same speckles. The identity of this velocity vector, along with the direction to Mercury, defines the plane that is perpendicular to that component of Mercury's spin vector that is instantaneously perpendicular to the line of sight to Mercury. An assumption of the Cassini state or two separate determinations of the speckle velocity vector define the obliquity, and instantaneous rotation rates are defined by the time delays necessary to maximize the cross correlation. The average speckle size is only a few kilometers, and the correlation becomes small if either antenna is more than a single speckle diameter off of the speckle velocity vector that passes through the other antenna. The uncertainty in the obliquity corresponding to this offset can be reduced by determining accurately the epoch of the correlation maximum, and the uncertainty in the rotation rate is similarly minimized by determining accurately the time delay for the correlation maximum. There are a sufficient number of antennas around the world that there are many occasions where various baselines become coincident with the speckle velocity vector at each conjunction. The ultimate precision in the obliquity and rotation rate depends on the baseline, radar wavelength, and integration time, but theoretical estimates of the uncertainty are typically near 1 arcsec in both the direction of Mercury's spin axis and in the amplitude of the physical libration (Holin, 2001). This technique has yet to be tested, so practical observational problems may increase these predicted uncertainties.

It is likely, with adequate tracking, that MESSENGER will determine C_{20} and C_{22} with a fractional uncertainty better than 0.001 along with fractional uncertainties in the obliquity θ of better than 1% and in the physical libration amplitude ϕ_0 of better than 8% (Zuber and Smith, 1997). The maximum fractional uncertainty in C_m/C in Eq. (34) is then reduced to ~9% yielding quite tight constraints on the core properties. If the radar experiments are successful in further reducing the uncertainties in θ and ϕ_0 to <1%, we would know C_m/C to ~2% (Eq. (34)), and we have potentially very tight constraints on the properties of Mercury's core—whether solid or molten. If accurate and secure radar results are obtained before MESSENGER arrives at Mercury, the *a priori* knowledge will decrease the uncertainty in the MESSENGER determinations and the combination of MESSENGER and radar data may lead to fractional uncertainties in C_m/C even less than 2%. This estimate in the uncertainty is at most doubled by the possible contribution of a large solid inner core to C_{22}. MESSENGER alone can tell us whether Mercury's outer core is molten and can therefore account for a fluid dynamo, but MESSENGER plus appropriate radar observations have the potential for further improving the details of core structure.

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