An experimental study of incremental surface loading of an elastic plate: Application to volcano tectonics

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Abstract. Models of surface fractures due to volcanic loading of an elastic plate are commonly used to constrain the thickness of planetary lithospheres, but discrepancies exist in predictions of the style of initial failure and in the nature of subsequent fracture evolution. In this study, we perform an experiment to determine the mode of initial failure due to the incremental addition of a conical load to the surface of an elastic plate and compare the location of initial failure with that predicted by elastic theory. In all experiments, the mode of initial failure was tension cracking at the surface of the plate, with cracks oriented circumferential to the load. The cracks nucleated at a distance from load center that corresponds to the maximum radial stress predicted by analytical solutions, so a tensile failure criterion is appropriate for predictions of initial failure. With continued loading of the plate, migration of tensional cracks was observed. In the same azimuthal direction as the initial crack, subsequent cracks formed at a smaller radial distance than the initial crack. When forming in a different azimuthal direction, the subsequent cracks formed at a distance greater than the radial distance of the initial crack. The observed fracture pattern may explain the distribution of extensional structures in annular bands around many large scale, circular volcanic features.

Introduction

Axisymmetric surface loads, most notably volcanoes but also lava-flooded mare basins, constitute distinctive sources of regional stress on the Earth and other terrestrial planets. When stresses associated with loading are sufficient for lithospheric failure, the resulting tectonics contain information on the loading process and the structure of the lithosphere. For example, the spatial positions of tectonic structures have been utilized as constraints on the thickness of the mechanical lithosphere in the vicinity of surface loads on the Moon, Mars, and Venus [Melosh, 1978; Comer et al., 1985; Solomon and Head, 1980; Solomon et al., 1993]. Fig. 1 shows circumferential extensional structures that developed around the Martian volcano Pavonis Mons.

While studies of tectonic features in the vicinity of volcanoes and mare basins represent a common approach to understanding regional lithospheric structure, model predictions of deformation structures that form in response to loading are often not consistent with observations. For example, Fig. 2a shows that if an assumed lithosphere initially fails by faulting, consideration of orientations of principal stresses [Anderson, 1951] results in a prediction of radially-oriented thrust faults (RT) under the load which transition to strike-slip faults (SS) and circumferential normal faults (CN) with increasing radial distance outside the load. However, if the least compressive principal stress component at a given distance from the load center has a magnitude that is much greater than the shear stress resolved along a given plane, the lithosphere will fail in tension rather than shear. For the axisymmetric loading problem, Fig. 2a shows that the least compressive stress occurs in the radial direction at a distance where the other two stress components are negligible, indicating that circumferentially oriented vertical tensile cracks should form outside the load (Fig. 2b).

Models of axisymmetric surface loading that assume initial deformation of the lithosphere by shear failure predict annular zones of strike-slip faulting outside the loads that are not observed in images of planetary surfaces [e.g., Melosh, 1976, 1978; Comer et al., 1985; Golombek, 1985]. Several studies have discussed the discrepancy between models and observations. Possible explanations include nucleation of faults at subsurface interfaces [Golombek, 1985], the consideration of stress magnitudes as well as directions in...
predictions of initial failure [Schultz and Zuber, 1994], explicit modeling of incremental growth of the volcanic load so that model stresses are not greatly in excess of that required for lithospheric failure [McGovern and Solomon, 1993], and nucleation of micro-scale strike-slip faults that transition to normal faults at macro-scales due to plastic yielding effects [J. Melosh, pers. comm., 1994]. However, studies that include these approaches still result in different predictions as to whether the mode of initial lithospheric failure is shear or tensile [McGovern and Solomon, 1993; Schultz and Zuber, 1994]. It is further difficult to address how deformation develops with time because after initial fracture the plate is not homogeneous, so elastic plate theory can no longer be used to predict modes of failure [Pollard and Segall, 1987]. In order to better understand the tectonic consequences of volcanic loading of the lithosphere and to provide guidance for future models of this process, we have performed an experiment in which we incrementally impose a conical surface load on an elastic plate. We observed the mode and location of first failure and investigated the development of deformation with continued loading.

Experimental Approach

Previous analytical treatments of the lithosphere loading problem [e.g. Melosh, 1978; Comer et al., 1985; Solomon and Head, 1980; Solomon et al., 1993] have assumed a uniform, elastic plate, which is a simplification of the depth distribution of strength in the lithosphere as indicated by laboratory experiments [Byerlee, 1968; Kirby and Kronenberg, 1987]. To compare our experiment to previous results, we also loaded an elastic plate. For the purposes of the experiment, it was necessary for the plate to have a low enough strength near the surface so that it would fail under gravitational loading, yet a high enough strength at its base to support loads. The elastic plate was made using a combination of agar and sand for the top part of the plate and a mixture of gelatin and sand for the lower part. Grade A Agar, distributed by Baxter Diagnostics Inc., is a chemical polymer that exhibits linear elastic properties over the range of conditions of interest [Lister and Kerr, 1991] and is relatively weak in cohesion, while high-clarity, 250-bloom, pig skin-derived gelatin by Kind and Knox Company is relatively strong in cohesion for the same degree of stiffness. Both agar and gelatin have previously been used to model crack propagation in the lithosphere [e.g. Takada, 1990; Lister and Kerr, 1991]. Sand composed of 99% quartz grains with an average grain size of 0.35 mm was used to increase the stiffness of the plate relative to its tensile strength. The elastic plate was supported on a weak viscous substrate of corn syrup and was incrementally loaded with lead shot.

To prepare the experiment, a layer of corn syrup approximately 8 cm deep was poured into a glass tank with dimensions 30 x 27 x 18 cm and the apparatus was chilled to make the syrup viscous enough to support the emplacement of the plate. The bottom of the plate was prepared by making a 3% concentration solution of gelatin, mixing in dry sand, and spreading it over the corn syrup. After refrigeration to allow
the gelatin to set, the top of the plate was prepared by spreading a 1% concentration solution of agar mixed with dry sand over the gelatin and sand plate. The whole set-up was again refrigerated to allow the agar to gel. In both parts of the plate, the weight ratio of sand to water was approximately 3.7 to 1. The two above mixtures formed a horizontally homogeneous elastic plate with a thickness of about 1 cm.

After the syrup was allowed to warm for about 10 minutes, a load of 100 grams of lead shot was added in the shape of a short cone. While analytical solutions use a gaussian load [Melosh, 1978], a conical load was easier to work with in the experiment. The difference between the shape of a cone versus a gaussian load is a second order effect [Comer et al., 1985] that does not affect the plate-scale stress state at the scale of interest. After 40 minutes, the load was incrementally increased by adding 30 grams of shot every 10 minutes thereafter. The plate began to bend shortly into the experiment and after approximately 80 minutes, the load was sufficiently large that the elastic stresses due to bending exceeded the tensile strength of the plate and the plate began to fail through the formation of cracks. The cracks in the plate were dyed with food coloring so they could be more easily identified in photographs (Fig. 3a).

After the cracking ceased and was recorded, samples of the plate were cut from the top of the corn syrup so that the Young's modulus (E) of the plate material could be measured. To measure E, a beam of the plate was supported between two points and allowed to bend under its own weight. The vertical deflection of the beam was measured using a high-frequency induction coil proximity probe (model KD-2400 manufactured by Kaman Sciences Corporation) and from this, E was calculated using the method described by Turcotte and Schubert [1982] equations for stresses due to axisymmetric loading of an elastic plate over a weak viscous substrate. For applicable experimental parameters, we calculated normal stress components as a function of distance from the center of the load for each experiment. Table 1 demonstrates that in each case, the distance from the center of the load where failure occurs corresponds to where the calculated maximum radial stress, \( \sigma_{rr} \), has the greatest tensile magnitude (e.g. Fig. 2b). Errors in experimental measurements caused some scatter in the predicted location of initial failure, as indicated by the shaded bar in Fig. 4 and by the range of values quoted for \( r/a_{\text{Calc}} \) in Table 1. However, the experimentally observed location agrees well with theory (Fig. 4) and indicates that the position of maximum \( \sigma_{rr} \) is a useful indicator of initial failure.

An advantage of our experimental approach is that it was possible to investigate how deformation progressed after initial failure, when elastic plate theory no longer applies. After the initial fracture, remaining stress in the plate resulted in an increase in the displacement across the initial crack. This was accompanied by the crack propagating in a mode I style around the load at the same radial distance.

The emplacement of additional load sometimes produced cracks which formed after the initial crack. The locations of these secondary cracks depend on the azimuthal direction of the crack with respect to the initial crack and is illustrated in Fig. 5. When the secondary crack formed in the same azimuthal direction with respect to center as the initial crack, it formed at a radial distance less than that of the initially formed crack (case a). The stress field in this direction is strongly affected by the initial failure and stresses due to additional loading are not felt by the region outside the initial failure. When the additional crack formed in a different azimuthal direction than initial failure, it formed at a greater radial distance (case b). The elastic nature of the plate in a

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**Table 1. Observed and Calculated Radial Distances of Initial Failure**

<table>
<thead>
<tr>
<th>Experiment</th>
<th>( r/a_{\text{Obs}} )</th>
<th>( r/a_{\text{Calc}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.94</td>
<td>3.81</td>
</tr>
<tr>
<td>2</td>
<td>3.64</td>
<td>3.3-3.75</td>
</tr>
<tr>
<td>3</td>
<td>3.75</td>
<td>4.0</td>
</tr>
<tr>
<td>4</td>
<td>4.36</td>
<td>4.6-4.8</td>
</tr>
<tr>
<td>5</td>
<td>3.91</td>
<td>4.3-4.5</td>
</tr>
</tbody>
</table>

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**Figure 4.** Radial stress \( \sigma_{rr} \) versus \( r/a \) for Expt. #2. \( \sigma_{rr} \) was calculated by using experimental parameters in the elastic solution [Melosh, 1978] and is plotted versus \( r/a \) to show the distance from load center where the maximum radial stress occurs. The range of predicted initial failure is shown by the bar to be \( r/a = 3.3-3.75 \). The observed initial failure occurred within the predicted range at \( r/a = 3.64 \).
distribution of deformation in an annular zone surrounding the different azimuthal direction than initial failure is not as strongly affected by the initial failure. As the radius of the load is increased, the radial distance of calculated maximum radial stress increases in azimuthal directions where plate failure has not occurred. This results in an outward migration of cracks in these directions. Sweeping out of deformation with time as the load increased was suggested by Comer et al. [1985] in a study of fractures around major Martian shield volcanoes, but was not based on quantitative calculation of loading of an already-fractured plate. The observed distribution of deformation in an annular zone surrounding the load may explain distributed concentric fractures around volcanic features, such as is shown in Fig. 1 [Comer et al., 1985; McGovern and Solomon, 1993].

Conclusions
The results of our experiment show that an incrementally loaded elastic plate initially fails by circumferential cracking at a distance from the center of the load that is consistent with theoretical calculation of maximum radial stress $\sigma_{rr}$. Further loading of the plate subsequent to the formation of the initial crack produces cracks at different radial distances than the initial failure. At no point in any of the experiments was strike-slip motion at a scale larger than sand grains observed either in association with the cracks or elsewhere in the plate. We do not, however, rule out the possibility that shear deformation structures (e.g. circumferential normal faults) may develop with continued loading. Due to scale, the experiment does not reproduce all of the conditions associated with loading of the lithosphere, notably gravity and the depth-distribution of lithospheric strength. However, we have provided experimental verification for a theoretical formalism that has long been employed to determine the thickness of an elastic lithosphere given the radial distance of concentric extensional structures around surface loads [Solomon and Head, 1980; Comer et al., 1985].

Since concentric tension cracks form around a load when the criterion for tensile failure is met, the approach of utilizing a tensile failure criterion to predict the distance of failure from the surface load is appropriate for tectonic studies of the early stages of volcanic loading of the lithosphere. The formation of extensional deformation features at greater and smaller radial distances with continued loading indicates that temporal aspects of load emplacement may significantly influence the pattern of tectonic features. It has now been demonstrated theoretically [Schultz and Zuber, 1994] and experimentally that an elastic plate with an axisymmetric vertical load initially fails in tension. Future models which utilize deformation features around surface loads to investigate the nature of the lithosphere of a planetary body should take these results into account.

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References

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