The Shallow Structure of the Martian Lithosphere in the Vicinity of the Ridged Plains

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Wrinkle ridges in the smooth plains of the Coprates and Lunae Planum regions of Mars form linear or concentric patterns with a regular spacing of 25-50 km. We test the hypothesis that the periodic development of deformation was a consequence of unstable horizontal compression, that occurred prior to probable ridge-related faulting, of a strength-stratified lithosphere which consists of a mechanically strong surface plains unit that successively overlies a weak megaregolith and a strong lithospheric basement. Results show that a range of models with both rigid and deformable megaregolith-basement interface conditions yield solutions which can explain the ridge spacing within the constraint provided by the estimated thickness of the smooth plains materials. In models that incorporate viscous and plastic rheologies, uniform and exponentially varying vertical strength distributions, and the presence or absence of interfacial slip at the base of the surface plains unit, the ridge spacing is primarily controlled by the power law exponent of the lithosphere, the megaregolith/plains unit thickness ratio, and the plains unit/megaregolith strength contrast. Deformable basement models with a viscous rheology (1 ≤ n ≤ 3) can explain the ridge spacing if the megaregolith was thicker than and approximately 1-3 orders of magnitude weaker than the plains unit. Similar models with a perfectly plastic (n → oo) rheology require plains unit thicknesses greater than 5 km, which exceeds most observational estimates. The dominant wavelength in the plastic models is not sensitive to internal strength contrasts and provides no constraint on the relative competence of the plains and megaregolith. Ridge spacing in the combined viscous and plastic models can be consistent with either a dry, water-rich, or ice-rich megaregolith at the time of ridge formation. If the ridge spacing was controlled primarily by the mechanical properties of lithospheric basement with little or no influence from the plains unit, then the minimum megaregolith thickness in the vicinity of the ridges must have been of the order of 0.4 km. The fact that ridges are most commonly found in the smooth plains but ridge spacing can be explained by models without a plains unit suggests that the unit may have facilitated the nucleation of reverse faults that are interpreted to characterize ridge structure but conceivably could have folded passively in response to unstable compression of the basement.

INTRODUCTION

One of the fundamental pieces of information required for understanding the tectonic evolution of a planetary body is the mechanical structure of the lithosphere. On a regional scale, direct constraints on the mechanical structure can be obtained from the geometries of tectonic features on the surface. Wrinkle ridges are common tectonic features on Mercury, the Moon, and Mars and are frequently found in smooth plains units of these bodies [Strom et al., 1975; Trask and Guest, 1975; Schultz, 1976; Maxwell, 1978; Strom, 1979; Plescia and Golombek, 1986; Sharpton and Head, 1988; Watters, 1988a]. On Mars, the most prominent assemblage of plains ridges occurs in the Coprates and Lunae Planum regions in the smooth plains materials [Chicarro et al., 1985]. These ridges are believed to have formed due to compressional stresses associated with the response of the lithosphere to the Tharsis load [Banerdt et al., 1982; Maxwell, 1982; Sleep and Phillips, 1985; Watters and Maxwell, 1986]. Many of the ridges exhibit either a linear, parallel trend or a Tharsis concentric pattern (Figure 1) with a regular spacing in the range 25-50 km (Table 1) [Wise et al., 1979; Maxwell, 1982; Watters and Maxwell, 1986; T. R. Watters, Periodically spaced wrinkle ridges in ridged plains unit on Mars, submitted to Journal of Geophysical Research, 1990, hereinafter referred to as Watters, 1990]. In areas where the ridges display such patterns, their spacing can be used to constrain the structure of the martian lithosphere at the time the features formed.

In this study we develop and quantitatively evaluate a suite of tectonic models that relate the regular spacing of the plains ridges to the shallow internal structure of Mars. Our analysis consists of the following approach: We first formulate a range of general rheological models of the Martian lithosphere in the vicinity of the ridged plains. We then solve for relationships between ridge spacing and rheological structure using linear stability analysis for a horizontally compressing, multilayered medium. Finally, we use the observed spacings of the ridges in combination with estimates of plains unit thickness to constrain the vertical structure of the lithosphere. We demonstrate that a variety of simple mechanical models can theoretically explain the ridge spacing. We then consider several possible implications of the models for the structure and state of stress in the Tharsis region at the time of ridge formation.
BACKGROUND

Shallow Structure of Mars

The development of a meaningful mathematical representation of the near-surface vertical structure of Mars in the vicinity of the plains ridges requires careful consideration of current information on the composition and competence of the lithosphere, as well as on the depth distribution of stresses responsible for ridge formation.

Materials that comprise the ridged plains have been interpreted as basalt-like volcanic flows primarily on the basis of their morphologic similarity to lunar maria and the observed presence of flow fronts and wrinkle ridges [Carr, 1973; Scott and Carr, 1978; Greeley and Spudis, 1981; Scott and Tanaka, 1986]. On the basis of the lack of observed multiple flow lobes at unit boundaries, Greeley and Spudis [1981] have interpreted the ridged plains, in the classification of Walker [1972], as simple flows that represent high-volume, single cooling units. Watters (1990) interpreted the plains as layered, multiple volcanic flows on the basis of analogy to the terrestrial Columbia Plateau basalts and lunar maria. Another factor in support of a volcanic origin is that well-developed wrinkle ridges are also found within the calderas of many of the major Martian shields, where the fill material is almost certainly volcanic [Greeley and Spudis, 1981]. Observations of sinuous rilles in association with ridged plains in Syrtis Major [Schaber, 1982] and Lunae Planum [Mouginis-Mark et al., 1990] provide further support for a volcanic origin. However, depolarized radar data [Thompson and Moore, 1989] show that some areas of the plains

<table>
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<tr>
<td>West Lunae Planum</td>
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<td>East Lunae Planum</td>
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<td>and East Chryse Planitia</td>
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<td>2</td>
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<td>Coprates (domain C2)</td>
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<td>Coprates</td>
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<tr>
<td>Chryse Planitia</td>
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*1, Saunders et al. [1981]; 2, Watters [1988b]; 3, Watters [1988c].

Fig. 1. Examples of regularly spaced ridges in eastern Solis Planum (longitude, +82.3º; latitude, -24.9º). The ridges are located in ridged plains material. Viking orbiter frame 608A43.
follow from the assumptions that (1) rim heights of measured plains embay cratered terrain, and to the northwest in the fication of the ridged plains/substrate interface. 

personal communication, 1988] to greater than 2 km (Wat-ranges from approximately one [Nedell et al., 1987; Robin-son and Tanaka [1988]; 7, R. O. Kuzmin (personal communi-
tions of these techniques have yielded values in this range,
but the evidence is not totally conclusive. 

of highland terrain underlying the resurfacing material is
predate the onset of resurfacing, and (4) the elevation
measurements reflect regional averages, (3) all the measured
thicknesses of mare deposits on the moon by at least a factor
of 2 [Head, 1982]. Possible sources of error in the estimation of thickness using flooded craters follow from the assumptions that (1) rim heights of measured fresh craters have not markedly degraded, (2) isolated measurements reflect regional averages, (3) all the measured craters predate the onset of resurfacing, and (4) the elevation of highland terrain underlying the resurfacing material is roughly uniform. Independent estimates of Martian smooth plains thickness have been made where the unit is exposed on the walls of the Valles Marineris and Kasei Valles. In both areas the thickness has been measured directly and ranges from approximately one [Nedell et al., 1987; Robin-
son and Tanaka, 1988; Luchitta et al., 1990, R. O. Kuzmin, personal communication, 1988] to greater than 2 km (Waters, 1990). These estimates depend on the accurate identifica-
tion of the ridged plains/substrate interface.

Along the eastern boundary of Lunae Planum the ridged plains embay cratered terrain, and to the northwest in the Kasei Valles the plains overlie a friable, chaotic material [Tanaka, 1986; Robinson and Tanaka, 1988]. These observations suggest that the ridged plains are underlain by the megaregolith, a mantle of fragmental, brecciated, and perhaps unconsolidated material that formed as a consequence of impact bombardment and subsequent reworking by aeolian, fluvial, and possibly glacial transport. The megaregolith that underlies the plains unit formed over an extended period of martian geologic history prior to the emplacement of the plains in the Early Hesperian [Tanaka, 1986]. Estimates of the thickness of the megaregolith are not well constrained and vary from hundreds of meters to as high as approximately 10 km, with most recent values in the vicinity of 2–3 km [Fanale, 1976; Kuzmin, 1983; Carr, 1986; Fanale et al., 1986; MacKinnon and Tanaka 1989; Woronow, 1988]. The competence of the megaregolith likely increases with depth, with brecciated and unconsolidated materials at shallow depths grading to less fractured materials with a strength that approaches that of coherent basement at greater depths.

Beneath the megaregolith lies basement, which we inter-
pret as the upper section of the mechanical lithosphere. (We define the mechanical lithosphere as the crust plus that part of the upper mantle which is capable of supporting appre-
ciable stresses over geological time scales.) Where exposed in the Kasei Valles, the depth to basement underlying the plains and megaregolith is observed to range from 1 to at least 3.5 km [Robinson and Tanaka, 1988]. Evidence from the Valles Marineris suggests that the transition between the megaregolith and basement is gradual because in most areas an abrupt transition from one to the other is not observed [Luchitta et al., 1990]. Estimates of the thickness of the mechanical lithosphere in areas without major volcanic constructs range from about 100 to 400 km [e.g., Lambeck, 1979; Turcotte et al., 1981; Banerdi et al., 1982; Willemann and Turcotte, 1982], while estimates in the vicinity of major volcanoes can be as low as 20–50 km [Comer et al., 1985]. In all cases the lithosphere is considerably thicker than the plains materials and megaregolith.

Stresses responsible for the formation of the regularly spaced plains ridges are thought to be associated with the formation of the Tharsis province [Banerdi et al., 1982; Maxwell, 1982; Sleep and Phillips, 1985]. Tharsis rises approximately 10 km above its surroundings and encum-
passes approximately a quarter of the surface area of Mars. Thus Tharsis-related stresses are global scale in nature. Global stress models have not specifically addressed details of the depth distribution of the stresses responsible for the development of the high topography or the tectonic features in this area. However, surface stress magnitudes predicted by these models, in combination with the depth variation of lithospheric strength on Mars estimated from strength enve-
lopes [e.g., Banerdi et al., 1990], suggest that stresses in the Tharsis region may have been present throughout much of the lithosphere.

The above information indicates that a generalized base-
line model for the Martian lithosphere in the vicinity of the ridged plains should consist of a thin, competent surface plains unit of probable volcanic origin that successively overlies a thin, incompetent region that corresponds to a weak megaregolith and a much thicker, competent region that corresponds to the upper part of the mechanical litho-
sphere. The models should also incorporate a range of vertical strength stratifications so that the depth distribution

<table>
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<th>TABLE 2. Thicknesses of Ridged Plains</th>
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<td>Memnonia–Phoenicis</td>
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<td>Sinai Planum</td>
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1 Estimates revised downward by DeHon [1988].
of compressional stresses responsible for ridge initiation can be investigated.

Ridge Spacing and Lithosphere Structure

A number of previous attempts have been made to explain the regular spacing of the plains ridges. Saunders and Gregory [1980] and Saunders et al. [1981] applied the theory of folding in a viscoelastic medium [Biot, 1961] and concluded that the spacing was controlled by the thickness of a competent or strong volcanic surface layer and the viscosity contrast between the volcanics and an incompetent or weak semi-infinite substrate that they interpreted as the megaregolith. On the basis of the wavelength/layer thickness ratio predicted by the theory and the thickness of the presumed volcanics, they determined that the viscosity contrast between the volcanics and the megaregolith was approximately 500. However, their model inherently assumed that the thickness of the megaregolith at the time of ridge formation was a significant fraction of the wavelength of the ridges. This assumption is not supported by megaregolith thickness estimates.

Watters [1986] suggested that ridge spacing was controlled by a 15-km-thick competent layer at an unspecified depth in the lithosphere. His suggestion was based on application of McAdoo and Sandwell's [1985] elastic-plastic buckling model under the assumption that the ridges are morphologically analogous to seafloor folds in the Indian Ocean. However, McAdoo and Sandwell's model requires plastic yielding at the top and bottom of the oceanic lithosphere strength envelope to reduce the elastic thickness, and therefore the buckling stress, to levels that can be consistent with folding deformation. Yielding will not be significant in the much thinner Martian plains unit. In addition, while the purpose of Watters' [1986] study was simply to demonstrate that the ridges were a consequence of upper crustal deformation, rather than to correlate the strong layer to any particular part of the lithosphere, we note that a strong layer of the order of 15 km thick is difficult to reconcile in the context of current knowledge of Martian lithosphere structure.

Watters [1990] alternatively proposed that the ridge spacing was controlled by an elastic surface plains layer that successively overlies an elastic megaregolith and a rigid lithospheric half-space. This model can explain the ridge spacing for a limited range of plains unit thicknesses and requires that the plains consisted of numerous interbedded flows that freely slipped at the interlayer contacts but were welded at the layer/megaregolith interface. However, it has not been established whether or not the lithospheric half-space could have been involved in the deformation. In addition, if the unit is not interbedded as assumed or was characterized by nonvanishing shear stresses at the interlayer contacts, then the critical stress required for deformation in his model would likely have exceeded the brittle strength of rock.

A possibility not considered in other studies is that ridge spacing was influenced by the coupled deformational properties of multiple competent regions within the lithosphere. Interpreted in the context of our baseline representation of Mars' shallow mechanical structure, these regions would most likely correspond to the surface plains unit and lithospheric basement. The observation that the ridge spacing (Table 1) is so much greater than the range of possible thicknesses of the plains unit (Table 2) suggests yet another possibility. Because ridge spacing must have been influenced by the mechanical properties of a vertical lithospheric section with a thickness much greater than that of the plains, the spacing conceivably could have been controlled primarily by the mechanical properties of lithospheric basement, with little or no influence from the surface plains unit [Zuber and Aist, 1988, 1989].

Model Formulation

The narrow horizontal cross section of the ridges in comparison to their spacing argues for a mechanism to localize deformation. A likely mechanism, based on morphological comparisons of the martian ridges to terrestrial and lunar ridges [Luchitta and Klockenbrink, 1981; Plescia and Golombek, 1986; Watters, 1988a], is reverse faulting. In the absence of preexisting, regularly spaced heterogeneities in lithospheric material properties, we assume that deformation prior to faulting controlled the sites of fault nucleation. This has been suggested by Watters [1988a] for the Martian ridges. We model the initiation of the deformation using a continuum approach, in which the spacing of the ridges can be determined from the dominant wavelength that develops due to the growth of compressional instabilities. The instabilities grow due to the magnification of randomly distributed, small-amplitude perturbations along interfaces between layers of different mechanical properties.

Determination of the dominant wavelength entails solving the Navier-Stokes equations for plane, quasi-static flow for a compressing, rheologically stratified medium. Because we are concerned with only the first increment of deformation, i.e., that prior to faulting, when the length scale of deformation is established, we require that the solutions obtained using this method must be valid to first order in the perturbing flow. The dominant wavelength, which is controlled by the thicknesses of the strong layers and other mechanical properties of the medium [cf. Fletcher, 1974; Smith 1975], is determined by the wave number k or wavelength A at which the dimensionless growth rate q of the instability is maximized. For uniform horizontal compression, the growth rate is related to the vertical amplitude of a random perturbation, \( \Delta(k, t) \), at the ith interface at time t by

\[
\Delta(k, t) = \Delta(k, 0) \exp \left[ (1 + q)\varepsilon_{xx} t \right] \tag{1}
\]

where \( \Delta(k, 0) \) is the amplitude of the initial perturbation and \( \varepsilon_{xx} \) is the mean horizontal strain rate. Compressional instabilities resulting in deformation with a dominant wavelength develop when the exponential term in equation (1) is greater than unity, in which case the magnitude of the initial perturbation amplifies with time.

This approach to modeling the development of periodic length scales of lithospheric-scale compressional deformation has previously provided useful information on the formation of wedge faults and duplex structures in terrestrial continental lithosphere [Davies and Fletcher, 1986], the structure of the oceanic lithosphere in the vicinity of intra-plate fold structures in the Central Indian Basin [Zuber, 1987a], and the structure of the Venus lithosphere in regions where compressional tectonic features are found [Zuber, 1987b; Zuber and Parmentier, 1990].

We explore a broad range of possible deformational mod-
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MODELS

Fig. 2. Models of the shallow Martian lithosphere examined in this study (top), shown with schematic profiles illustrating strength \( \tau \) as a function of depth \( z \) (bottom). The subscripts 1-4 correspond to the surface plains unit, megaregolith, lithospheric basement, and deep lithosphere, respectively. Model variables are defined in Table 3, and dimensionless parameters are defined in Table 4.

STRENGTH PROFILES

<table>
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<th>TABLE 3. Model Variables</th>
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<tr>
<td>Variable</td>
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<td>( z )</td>
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<tr>
<td>( \lambda_d )</td>
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<td>( h )</td>
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<td>( \rho )</td>
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<td>( \tau )</td>
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...
For small megaregolith thicknesses these parameters trade off such that a larger value of $R_1$ is focused on the general behavior of the model. Note that a given $Zuber [1987b].$

However, for large megaregolith thicknesses the plains unit does not “feel” the effect of the substrate; the dominant wavelength is controlled primarily by the plains unit thickness and the plains/megaregolith strength contrast. At large $h_2/h_1$, deformation in the plains unit is effectively decoupled from that in the basement.

From Figure 3 we seek a family of solutions that can explain the ridge spacing. The observed range of spacings ($25 \leq \lambda \leq 50$ km) and plains unit thicknesses ($0.25 \leq h_1 \leq 2$ km) indicate that solutions with wavelength/plains unit thickness ratios of $12.5 \leq \lambda_1/h_1 \leq 200$ are theoretically allowable. The minimum value of $R_1$ compatible with this range of $\lambda_1/h_1$ occurs in the limit of large $h_2/h_1$ and is approximately equal to 10. The upper limit of $R_1$ can be estimated from measured strengths of plains unit- and megaregolith-like materials. In the absence of ground truth data for Mars, we consider existing measurements of material properties for analog materials for the Moon and Earth. $P$ wave velocities have been determined from the Apollo 17 Lunar Seismic Profiling Experiment (LSPE) and show a maximum velocity contrast between the surface megaregolith and possibly fractured bedrock in the shallow lunar subsurface (depth $\approx 2$ km) of the approximately a factor of 50 [Kovach and Watkins, 1973; Cooper et al., 1974]. This translates to a maximum contrast in Young’s modulus between the lunar megaregolith and basement of just under 2 orders of magnitude by assuming that $S$ wave velocity is 0.6 times the magnitude of the $P$ wave velocity [cf. Golombok, 1985]. Elastic constants of lunar basalts and breccias have also been measured in the laboratory and show an order of magnitude or less difference [Warren and Trice, 1977; Trice et al., 1974]. On the Earth, contrasts in compressional differential stresses at constant strain between unfractured basalt and unconsolidated sands or clays can be considerably in excess of 2 orders of magnitude [Clark, 1966]. In the case of Mars, higher strength contrasts are possible if the megaregolith was water- [Carr, 1986] or ice-rich [Squyres and Carr, 1986] at the time of deformation, with the ice-rich case more relevant to the assumption of linear viscosity in the current model. If we require $R_1 \approx 1000$, then solutions in which $h_2/h_1 \approx 1.5$ are consistent with the ridge spacing.
Rigid Basement Model

The conclusion that \( h_2/h_1 \to 1 \) is valid for all \( R < 5000 \). unit to explain the ridge spacing would remain unchanged. However, our conclusion would be implied. If the lower part of the range of \( h_2/h_1 \) and vice versa. If the maximum value of \( R_1 \) is smaller or larger than we assumed, then a larger or smaller minimum megaregolith thickness, respectively, would be implied. However, our conclusion that the megaregolith must be thicker than the surface plains unit to explain the ridge spacing would remain unchanged. The conclusion that \( h_2/h_1 \geq 1 \) is valid for all \( R_1 \leq 5000 \).

Rigid Basement Model

Next we assess the effect of the strength of the basement in the determination of the dominant wavelength. Figure 4 plots \( k'_d \) and \( \lambda_d/h_1 \) as a function of \( h_2/h_1 \) for a range of values of the strength contrast between the plains unit and basement \( (R_2 = \tau_1/\tau_3) \). Here we have assumed \( R_1 = 100 \) and other parameter values to be the same as Figure 3. The diagram illustrates that for small \( h_2/h_1 \), the dominant wavelength decreases as the ratio of plains unit/lithospheric basement strength \( (R_2) \) decreases. The solution achieves limiting behavior when the strength of the basement is a factor of 10 greater than that of the plains unit \( (R_2 \approx 0.1) \). In the limit of a strong basement, vertical and horizontal velocities in the basement vanish, and the megaregolith/basement interface is effectively a rigid boundary. Lithospheric deformation in the limit is “thin-skinned,” i.e., confined to the plains unit and megaregolith. This scenario corresponds to model 2. Figure 4 shows that for small \( h_2/h_1 \), the effect of a strong lower boundary is to shift the dominant wavelength to a smaller value than would be obtained if the boundary was deformable. Until limiting behavior is reached, a progressively stronger basement results in a progressively shorter dominant wavelength. In other words, for similar rheologies and plains unit thicknesses, the rigid basement model requires larger megaregolith thicknesses and/or plains/megaregolith strength contrasts than the deformable basement model to explain the ridge spacing. The effect is greatest for small \( h_2/h_1 \) because the mechanical properties of basement most significantly influence the pattern of surface deformation when the basement is at shallow depths.

For the parameters assumed in Figure 4, the solutions begin to deviate at \( h_2/h_1 < 2 \) but yield markedly different dominant wavelengths only for \( h_2/h_1 < 1 \). However, in the previous section we showed that solutions in the latter range could not explain the observed ridge spacing because they required unrealistically high internal strength contrasts. Because the dominant wavelength in the range \( h_2/h_1 \geq 1 \) is largely insensitive to the deeper lithospheric strength stratification, we conclude that even if the rheology and plains/megaregolith strength contrast were well constrained, it would not be possible to distinguish between shallow and deep stress penetration in the vicinity of the ridges solely on the basis of theoretical models of ridge spacing. We return to this point later in the analysis.

Variable-Strength Megaregolith Model

In the third model the lithosphere consists of a surface plains layer of uniform strength that overlies a semi-infinite substrate. The strength at the top of the substrate is less than that of the layer and increases exponentially with depth. The boundary conditions at the surface and at the base of the plains layer are the same as for models 1 and 2. This model corresponds to a scenario in which the megaregolith and lithospheric basement are not separated by a discrete boundary, as in models 1 and 2, but rather represent a continuum in which the megaregolith becomes increasingly compacted and/or less brecciated with depth and gradually grades into lithospheric basement, as observed by Luchitta et al. [1990] in the Valles Marineris. As for model 1, the basement is capable of deforming in response to the imposed horizontal compression. The relevant dimensionless parameters for this model are the strength contrast at the interface between the plains unit and the top of the substrate \( (R_1) \), and the \( e \)-folding thickness of the substrate strength \( (\xi) \), the latter of which constitutes a measure of the megaregolith thickness. The mathematical formulation for this model is detailed in the appendix.

Figure 5 shows \( k'_d \) and \( \lambda_d/h_1 \) versus the megaregolith/plains unit thickness ratio \( \alpha (= \xi/h_1) \) for a non-Newtonian viscous lithosphere \( (n = 3) \) and the limiting case of a strong surface layer in which buoyancy forces do not play a significant role in influencing the pattern of deformation \( (S = 0) \). As for the deformable and rigid basement models, \( R_1 \) and the normalized megaregolith thickness \( (\alpha) \) trade off in the determination of \( \lambda_d/h_1 \) so a wide range of lithospheric structures can explain the ridge spacing. If we apply the same observational constraints as invoked in model 1, then model 3 can be consistent with the ridge spacing if \( \alpha \geq 1 \) and \( 50 \leq R_1 \leq 1000 \). Given that differences in the dominant wavelengths predicted for Newtonian (assumed in models 1 and 2) and non-Newtonian (assumed in model 3) rheologies are minimal, models 1, 2, and 3 predict a similar lower limit of megaregolith thickness for the strong plains unit case \( (S = 0) \) if the \( e \)-folding thickness of the substrate \( (\xi) \) in model 3 is assumed to be analogous to the megaregolith thickness \( (h_2) \) in models 1 and 2.

Figure 6 illustrates results for model 3 for the limiting case of a weak plains unit \( (S = 10) \) in which buoyancy forces...
significantly influence the pattern of unstable flow. Figure 6 shows that a model with a weak plains unit can be consistent with the ridge spacing for a smaller lower limit of the e-folding thickness ($\alpha \geq 0.5$), and thus a thinner megaregolith, than the strong plains unit model. For large e-folding thickness ($\alpha \approx 12$), corresponding to a thick megaregolith, the medium is stable in compression; that is, deformation occurs by uniform horizontal shortening and a dominant wavelength does not develop. Thus lithospheric structures with a weak plains unit and a thick megaregolith ($\alpha \approx 12$) are unacceptable.

**Surficial Megaregolith Model**

In the fourth case the lithosphere structure consists of a weak surface layer that overlies a strong layer and a weaker half-space. The model corresponds to a weak, surficial megaregolith layer that is successively underlain by a strong lithospheric layer and a weaker lithospheric substrate. The boundary conditions at the surface are the same as in models 1–3, and stresses and velocities at both subsurface interfaces are assumed to be continuous. The model represents a situation in which either the surface plains unit does not exist or is so thin or weak in comparison to the lithosphere that it does not significantly influence the characteristics of deformation. The ridge spacing is controlled almost entirely by the thickness and mechanical properties of the basement. Deformation of the plains unit in this case would occur by passive folding in response to compressional instability of the mechanical lithosphere. Here we treat the lithosphere as a discrete layer rather than a half-space, as assumed in the other models, so that we can quantitatively assess the relationship between lithosphere thickness and ridge spacing. The mathematical formulation for this model is simplified from Zuber [1987b].

Results are summarized in Figure 7, which plots $k_d'$ and the dominant wavelength/megaregolith thickness ratio ($\lambda_d/h_2$) as a function of the lithospheric layer/megaregolith thickness ratio ($h_3/h_2$) for a range of values of the megaregolith/lithosphere yield strength contrast ($R_3 = \gamma_3/\tau_3$). (Note that parameters in this model are normalized by the properties of the megaregolith rather than the plains unit as was assumed for models 1–3.) The model assumes that the lithosphere has a perfectly plastic rheology ($n \to \infty$), which approximates a medium in which deformation occurs by pervasive faulting. Geological evidence that the megaregolith is composed of highly fractured materials and that faulting occurs in lithospheric basement, in combination with extrapolated experimental rock deformation data that indicate brittle behavior of the Martian near-surface, justify the use of this rheology. In this solution, as in other formulations of a deforming perfectly plastic lithosphere, the dominant wavelength is essentially independent of the magnitude of internal strength contrasts [cf. Zuber and Parmentier, 1986; Zuber, 1987a].

For small $h_3/h_2$, $\lambda_d/h_2$ varies considerably with lithosphere thickness; however, for $h_3/h_2 \geq 150$, $\lambda_d/h_2$ is essentially independent of this parameter. Given the maximum value of $\lambda_d/h_2$ and the range of observed ridge spacings, model 4 yields a minimum megaregolith thickness of 0.4 km.
Effect of Alternative Rheologies

In the solutions for models with a strong surface layer (models 1–3) discussed above, we assumed Newtonian ($n = 1$) or non-Newtonian ($n = 3$) viscous rheologies. We now investigate the effect of assuming instead a perfectly plastic rheology. In this case, $R_1$ represents the ratio of the yield strengths of the plains and megaregolith. Low values of $R_1$ are possible if both the plains and megaregolith were dry at the time of deformation. However, if the megaregolith contained liquid water [e.g., Carr, 1986], then it may have been significantly weaker than the plains due to the reduction in yield stress associated with increased pore fluid pressure, and high values of $R_1$ would be applicable.

Figure 8 illustrates results for model 1. Note that much smaller wavelength/plains unit thickness ratios characterize the deformation than in the viscous case, and consequently, larger plains unit thicknesses are required to explain the ridge spacing. Figure 8 indicates that the minimum plains unit thickness that can be consistent with the ridge spacing is approximately 5 km. Thus if the mechanical properties of the plains unit played a major role in controlling the ridge spacing and if the early stages of ridge formation were characterized by brittle deformation, then a plains unit thickness that exceeds most current estimates is required. In addition, as for model 4 above, the dominant wavelength is nearly independent of the magnitudes of internal strength contrasts. Therefore perfectly plastic lithosphere models cannot constrain the relative competence of the plains versus megaregolith at the time of ridge formation. In this case a dry, water-rich, or ice-rich megaregolith would have been possible.

Effect of Interfacial Slip

In all of the models discussed thus far, we have assumed that the interfaces between materials of contrasting rheological properties are welded; that is, stresses and velocities are continuous across them. However, because the Martian megaregolith, at least at shallow levels, is weak and possibly contains a significant component of fractured or unconsolidated materials, it is possible that the early stages of deformation related to ridge formation could have been accompanied by subsurface horizontal slip. If a slip plane developed, it likely would have done so at the depth where the horizontal shear stress is a maximum. For many solutions for models 1–3, this depth corresponds to the plains unit/megaregolith interface. To assess the potential importance of interfacial slip on the dominant wavelength, we have modified the rheological structure in model 1 to include free slip at the base of the plains unit and compared the solution to that for a welded interface. In the free slip case only the vertical normal stress and vertical velocity are continuous across the interface. Boundary conditions at other interfaces are the same as for model 1.

Figure 9 shows $k_d'$ and $\lambda_d/h_1$ as a function of $h_2/h_1$ for a range of $R_1$ for both free slip and welded interface conditions. The welded interface solution is reproduced from Figure 3. For small plains unit/megaregolith strength contrasts ($R_1 = 10$), the dominant wavelength that develops in the model lithosphere with the welded interface is greater than that with the freely slipping interface, but by less than about 10%. The difference between the solutions decreases as the megaregolith thickness ($h_2/h_1$) and strength ratio ($R_1$) increase. A similar relationship between dominant wavelength and strength contrast has been observed for folding of a single strong layer imbedded in a weaker half-space [Biot, 1959; Fletcher, 1977]. Figure 9 indicates that a slip plane at the base of the plains unit will influence the dominant wavelength minimally and only for small internal strength contrasts.

Summary of Models

An obvious outcome of the analysis is that a variety of simple mechanical models yield solutions that can explain the spacing of the plains ridges. In fact, none of the models that we have examined can be eliminated on the basis of ridge spacing alone. The general success of the models supports the hypothesis that ridge spacing was controlled by continuum folding of a strength-stratified lithosphere that occurred prior to the formation of faults believed to be associated with individual ridges.

The models exhibit common behavior in that the ridge spacing approached the ridge spacing for the free slip case as the strength contrast between the plains and megaregolith ($R_1$) increased.
spacing is primarily controlled by internal strength contrasts and the ratios of the thicknesses of strong and weak regions within the shallow lithosphere. For models of a viscous lithosphere with either a deformable or rigid basement and with or without horizontal slip at the base of the plains unit, we find that solutions with a strong surface plains unit in which the megaregolith is thicker than the plains unit, and with plains/megaregolith strength contrasts from approximately 1 to 3 orders of magnitude can be consistent with the ridge spacing. Viscous lithosphere models with a weak surface plains unit can explain the spacing for approximately the same range of plains/megaregolith strength contrasts for thinner megaregoliths. Perfectly plastic lithosphere models with a strong surface plains unit require a plains unit thickness of 5 km or more to explain the ridge spacing, while perfectly plastic lithosphere models without a surface plains unit can explain the spacing if the megaregolith has a thickness of approximately 0.4 km or more. The combined solutions can be consistent with a dry, water-rich, or ice-rich megaregolith at the time of ridge initiation.

These models demonstrate the importance of submegaregolith basement, which was neglected in previous models [Saunders and Gregory, 1980; Saunders et al., 1981; Watters, 1986], in controlling the relationships between ridge spacing, layer thickness, and internal vertical strength contrasts. Specifically, we find that a much broader allowable range of internal strength contrasts than determined by Saunders and Gregory [1980] and Saunders et al. [1981] can explain the ridge spacing. In addition, we dismiss the possibility of a 15-km-thick strong layer determined by Watters [1986]. We also find broader ranges of most model parameters than the layered plains unit models of Watters (1990), which, for arbitrarily assumed numbers and thicknesses of layer interbeds, required a plains unit thickness $\geq 2$ km, megaregolith thickness $\geq 1$ km, and plains/megaregolith strength contrast $\approx 1000$. The large minimum strength contrast in that model restricts the megaregolith to have been volatile-rich at the time of ridge formation, which is in contrast to our result that either a dry or volatile-rich megaregolith was possible. In addition, Watters’ model implies that observations of ridged plains thickness of $\leq 2$ km are not representative of the thickness in the vicinity of the regularly spaced ridges. In contrast, our models can be consistent with the entire range of measured plains unit thicknesses (Table 2).

For all models we have endeavored to define a wide range of acceptable solutions by consistently attempting to incorporate liberal uncertainties in model parameters and observational constraints and by examining a variety of rheologies and boundary conditions. In addition, we have allowed for uncertainties due to admitted approximations in our representations of Martian rheological structure. We note that because of the generality of our approach, our best fit parameters can be simply refined as better geological constraints become available from data returned from the planned Mars Observer and Mars 1994 missions.

**Other Implications for Martian Lithosphere Structure**

**Stress Magnitudes**

In the interest of examining end-member models, we presented solutions for the limiting cases of a strong ($S \to 0$) and weak ($S \to \infty$) surface plains unit. Some insight into the shallow stress levels associated with ridge formation can be obtained from investigating relationships between $\lambda_d$, $S$, $h_1$, and $\tau_1$. The relationship between $\lambda_d$ and $S$ is plotted in Figure 10 for model 1 for a range of $R_1$ and typical values of other lithospheric model parameters. Note that the solutions display limiting behavior corresponding to a strong plains unit for $S \approx 0.1$, and a weak plains unit for $S \approx 5$. Figure 11 is a plot of the strength of the plains unit ($\tau_1$) as a function of plains unit thickness ($h_1$) for a range of $S$ and two values of the density contrast at the surface ($\Delta \rho = \rho_1 - \rho_0$). A value of $\Delta \rho = 3000$ kg m$^{-3}$ (solid lines) corresponds to a surface plains unit composed of basalt, while $\Delta \rho = 800$ kg m$^{-3}$ (dashed lines) corresponds to a basaltic plains unit that is overlain by a thin, weak, regolith layer that maintains a level surface and has a density of 2200 kg m$^{-3}$. Corresponding curves for a plains unit of sedimentary origin, e.g., sandstone ($\rho = 2500$ kg m$^{-3}$; $\Delta \rho = 300$ kg m$^{-3}$), would be similar in shape but shifted to slightly lower strengths than those for basalt. Shown for comparison is the maximum compressive stress difference (dotted line) for rocks of arbitrary composition in the shallow Martian subsurface [cf. Byerlee, 1968]. At stresses in excess of the maximum compressive stress, fracturing is favored over folding. For the limiting cases in Figure 10, a strong surface plains unit corresponds to deformational stress levels of a few tens of megapascals, while a weak surface unit requires stresses on the order of just a few megapascals. Gravity-topography relationships predict radial compressional stresses in the periphery of Tharsis of the order of tens of megapascals [e.g., Sleep and Phillips, 1985]. Such stress levels are comparable to layer strengths in the strong layer models, though either strong layer model with $\Delta \rho = 800$ kg m$^{-3}$ or weak layer models with $\Delta \rho = 800$ or 3000 kg m$^{-3}$ can be theoretically consistent with the development of the periodic deformation.

The stresses estimated from the models do not permit us to distinguish between a volcanic and sedimentary origin for the ridged plains. Discriminating between these possibilities...
Depth Penetration of Stresses

In three of the four scenarios that we examined (models 1, 3, and 4), ridge spacing can be explained by a lithosphere in which stresses associated with uniform horizontal compression penetrate into basement. Calculations for the support of Tharsis topography by membrane stresses imply a stressed lithosphere with a thickness much greater than the thickness of the plains plus megaregolith [Willemann and Turcotte, 1982; Sleep and Phillips, 1985], which is consistent with our deforming lithosphere models. The plausibility of these models is also consistent with evidence that wrinkle ridges are observed in exposed basement in the Kasei Valles [Robinson and Tanaka, 1988], and with recent results of Golombek et al. [1989] that significant depth penetration of compressional deformation may be indicated by measurements of shortening across ridges in Lunae Planum. The latter analysis suggests that planar thrust faults, which are presumed to be characteristic elements of ridge structure, extend to depths of several tens of kilometers. In the context of our analysis, the locations of basement faults would have been controlled by preceding folding.

A model of a compressing lithosphere with a rigid basement (model 2), which corresponds to thin-skinned deformation, can also explain the ridge spacing. On Earth, thin-skinned tectonics is often associated with gravity sliding and generally occurs in response to regional stresses, rather than global stresses such as those responsible for Tharsis deformation [cf. Crittenden et al., 1980]. Topographic cross sections of a limited number of ridges in Lunae Planum have been derived using photoclinometry [Plescia, 1988]. Where they have been measured, north-south trending ridges have a higher elevation on the east, on the side away from the topographically high Tharsis dome [Golombek et al., 1989; B. Luchitta, personal communication, 1989], which is the opposite configuration expected if these features formed due to gravity sliding. However, this trend should be interpreted with caution as photoclinometry has inherent problems in determining regional elevation changes. Thin-skinned compression may be associated with other mechanisms such as compressive plate boundary forces on Earth [Davis et al., 1983] or proposed plate convergence [Head, 1988] or crustal delamination [Turcotte, 1989] on Venus. However, these mechanisms are unlikely to have produced thin-skinned tectonics on Mars. Nonetheless, the existence of periodically spaced ridges in areas spatially removed from the influence of Tharsis (Watters, 1990) suggests that at least some ridges could have formed due to shallower, regional-scale stresses.

Lateral Variations in Lithosphere Structure

Because few of the reported measurements of ridge spacing are accompanied by error estimates (cf. Table 1), it is difficult to confidently distinguish the extent to which published differences represent measurement uncertainties or actual spatial variations in spacing. Recognition of the latter would be indicative of lateral variations in lithosphere structure, e.g., regional differences in the thicknesses of the plains unit or megaregolith, or the magnitudes of internal strength contrasts. The possible variation of ridge spacing with plains unit thickness could, in principle, be evaluated, if a sufficient number of flooded impact craters can be identified in the vicinity of regularly spaced ridges.

The spatial occurrence of regularly spaced ridges has implications for lateral variations in lithosphere structure and the nature of applied compressional stresses. For example, periodically developed ridges are common to the east and southeast of Tharsis but occur less frequently north and west of the province. This could indicate an asymmetric Tharsis stress distribution, in which stresses in some areas do not achieve the magnitude required for compressional instability growth. Alternatively, the absence of regularly spaced ridges may reflect a lithospheric structure that did not permit ridge development. A possible example of such a structure is a region without a surface plains unit. While our work has shown that the ridge spacing can be explained by unstable compression of a lithosphere without a plains unit (model 4), we note that compressional ridges both on Mars and the other terrestrial planets occur in most instances in plains materials [Strom et al., 1975; Trask and Guest, 1975; Schultz, 1976; Maxwell, 1978; Strohm, 1979; Chicarro et al., 1985; Plescia and Golombek, 1986; Watters, 1988a]. This observation suggests that whereas the plains unit may not have been required for the establishment of the periodic length scale, it may have facilitated the nucleation of reverse faults that likely characterize ridge morphology.

Deeper Structure of the Lithosphere

Compression of a lithosphere that contains more than one strong layer can result in the development of multiple
wavelengths of deformation [Zuber, 1987b]. Banerdt et al. [1990] assumed a Martian lithosphere consisting of a basaltic crust over an olivine mantle with a thermal gradient of \( dT/dz = 9^\circ \text{K km}^{-1} \) and a horizontal strain rate of \( \varepsilon_{xx} = 10^{-15} \text{s}^{-1} \). They showed that if the crust is less than about 40 km thick, then the lithospheric strength envelope, which plots differential stress versus depth, will contain two strength maxima. However, the observation that ridges exhibit a single, albeit variable, length scale of deformation in Lunae Planum and Coprates indicates either the absence of multiple discrete strong layers in the upper few tens of kilometers of the Martian lithosphere or the presence of two strong layers with the lower layer weaker than the upper layer [Zuber, 1987b]. Interpreted in the context of the depth distribution of strength calculated by Banerdt et al. [1990], the absence of multiple shallow strong layers would be consistent with a crustal thickness of greater than 40 km in the vicinity of the regularly spaced ridges. A thinner or thicker crust would be indicated if the thermal gradient were higher or lower, respectively, than Banerdt et al. assumed. Bills and Ferrari [1978] determined the crustal thickness in the Lunae Planum area to be in the range 50–70 km from Bouguer gravity anomalies assuming a constant density, variable thickness crust, and a global mean crustal thickness of 40 km. A much greater average thickness of the crust, estimated on the basis of the crustal thickness beneath the Hellas Basin determined by Sjogren and Wimberly [1981], would imply a crustal thickness in the vicinity of the ridges of >150 km.

A much longer wavelength of deformation, possibly manifest as topographic undulations superposed on, trending parallel to, and formed contemporaneously with the ridges, might be expected if folding of the entire (crust plus mantle) or deep (mantle only) lithosphere accompanied ridge formation. This longer wavelength of deformation could have occurred if radially oriented Tharsis-related compressional stresses penetrated into the deep lithosphere and exceeded the critical magnitude required for lithospheric-scale folding. Our analysis has demonstrated the plausibility of stresses of sufficient magnitude to result in deformation of shallow lithospheric basement, but this study was not designed to address stress magnitudes in the lower lithosphere. If deformation of the deeper lithosphere occurred, then lithosphere thickness estimates [Lambeck, 1979; Turcotte et al., 1981; Banerdt et al., 1982; Willemann and Turcotte, 1982; Comer et al., 1985] and compressional deformation models [Zuber, 1987a, b] indicate that the dominant wavelength of folding could range from as low as 100 to well over 1000 km. Watters [1986] has suggested the existence of long wavelength compressional deformation from analysis of Earth-based radar-derived topography in southern Tharsis [Roth et al., 1980]. However, reliable detection of such long wavelengths will require long baseline topographic profiles that are accurately referenced with respect to Mars' center of mass. A global topographic field with the required long-wavelength integrity will be possible with the altimeter experiment on the Mars Observer mission [Smith et al., 1989]. The presence (or absence) of this long wavelength topography, and its wavelength and spatial occurrence, or the recognition of an alternative expression of long-wavelength periodic deformation of the surface or subsurface (e.g., gravity anomalies), would provide important information on the deep structure and stress state of the martian lithosphere in the Tharsis region.

**Conclusions**

We have investigated the regular spacing of wrinkle ridges in smooth plains of the Coprates and Lunae Planum regions of Mars through the development of mechanical models of lithospheric compression. We have shown that a range of models of a compressing, strength-stratified lithosphere can be theoretically consistent with the ridge spacing. Successful models include those in which the dominant wavelength of folding develops due to unstable compression of the surface plains unit, lithospheric basement, or both the plains unit and basement. In the first case the basement is rigid, which corresponds to a scenario in which stresses are confined to shallow depths, and is characterized by deformation of the plains unit and megaregolith. In the second and third cases, stresses penetrate into and deform basement as well as the plains unit and megaregolith. Models with basement involvement in the deformation are consistent with limited observational data as well as geophysical models that indicate a high stress levels in a thick lithosphere. Models in which basement is not involved in the deformation are supported by the existence of periodically spaced ridges in areas removed from global-scale, and presumably deeply penetrating, Tharsis stresses.

If the mechanical properties of the plains unit influenced the ridge spacing, then the spacing is a sensitive function of the lithosphere rheology and the relative thicknesses and strengths of the plains unit and megaregolith. For a viscous \( (1 < n \leq 3) \) lithosphere with or without horizontal slip at the base of the plains unit, the observed ridge spacing and the estimated range of plains unit thicknesses constrain the megaregolith to have been thicker than and approximately 1–3 orders of magnitude weaker than the overlying plains unit at the time of ridge formation. In similar models with a perfectly plastic \( (n \rightarrow \infty) \) lithosphere, the dominant wavelength is essentially independent of internal strength contrast and thus does not provide a constraint on the relative competence of the plains and megaregolith. These models predict much smaller wavelength/layer thickness ratios than the viscous lithosphere models and consequently require a plains unit thicknesses of 5 km or more, which exceeds observational estimates of this parameter. The combined viscous and plastic results are consistent with the existence of a dry, water-rich, or ice-rich megaregolith at the time ridge formation initiated.

If the spacing of the ridges was primarily controlled by the thickness and mechanical properties of lithospheric base-
Fig. A1. (a) Strength as a function of depth in a model lithosphere consisting a strong surface layer over a substrate in which the strength at the interface is equal in magnitude to that of the layer and decreases exponentially with depth. (b) Strength as a function of depth in a model lithosphere consisting a strong surface layer over a substrate in which strength is discontinuous at the layer-substrate interface and increases exponentially with depth. The parameters \( \gamma \) and \( \zeta \) are the e-folding thicknesses of the substrate in models in Figures A1a and A1b, respectively.

APPENDIX: SOLUTION FOR THE PERTURBING FLOW WITH DEPTH-DEPENDENT VISCOSITY OR STRENGTH

We wish to determine the perturbing flow in a compressing medium consisting of a layer of uniform effective viscosity \( (\mu) \) or strength \( (\tau = 2\mu \dot{e}_{xx}) \) underlain by a substrate in which strength increases exponentially with depth, and where strength is discontinuous across the layer-substrate interface. To gain insight into this problem we first examine a flow in which strength is continuous across the layer-substrate interface and decreases exponentially with depth in the substrate. The solution to such a flow is documented by Fletcher and Hallet [1983]. For this case, shown in Figure A1a, the depth dependence of strength in the substrate has the form

\[
\tau(z) = \tau_0 e^{z/\gamma}
\]

where \( \tau_0 \) is the strength of the overlying layer, \( \gamma \) is the e-folding length of the substrate strength, and \( z \) is depth measured negative downward. The equation for the flow in a medium is [Fletcher and Hallet, 1983]

\[
D^4W + 2\gamma^{-1}D^3W + [\gamma^{-2} - 2k^2(2\gamma - 1)]D^2W - 2(2\gamma - 1)\gamma^{-1}k^2DW + k^2(k^2 + \gamma^{-2})W = 0 \tag{A2}
\]

where \( D = dz/dz \), \( k = 2\pi/\lambda \) is the wave number and \( W \) is the stream function satisfied by

\[
\dot{w} = W \cos(kx)
\]

\[
\dot{u} = -k^{-1}DW \sin(kx) \tag{A3}
\]

where \( \dot{u} \) and \( \dot{w} \) are the horizontal and vertical components, respectively, of the perturbing velocity field. To model a substrate in which strength increases exponentially with depth and where a strength discontinuity exists at the layer-substrate interface, as shown in Figure A1b, we let the strength in the substrate satisfy

\[
\tau(z) = \tau_1 e^{z/\zeta}
\]

where \( \tau_1 \) is the strength of the substrate at the interface, \( \zeta \) is the e-folding length of the substrate strength, and \( z \) is measured positive downward. Hence the flow in the exponentially increasing strength model also satisfies equations (A2) and (A3) if \( \tau_0 \) is replaced by \( \tau_1 \) and \( \gamma \) by \( -\zeta \). With these changes, (A2) has the general solution

\[
W(k, z) = [B_1 \cos(1/kz) + B_2 \sin(1/kz)]e^{\alpha'kz}
\]

\[
+ [B_3 \cos(1/kz) + B_4 \sin(1/kz)]e^{\alpha''kz} \tag{A5}
\]

where

\[
\alpha' = a - m/2
\]

\[
\alpha'' = -a - m/2
\]

\[
\beta = r/a
\]

\[
r = [m^2/4 + (n_2 - 1)/n_2^2]^{1/2}
\]

\[
a = (2^{1/2}/2)(m^2/4 + 2/n_2 - 1 + [m^4/16 + (m^2/2)(2/n_2 + 1) + 1]^{1/2})
\]

\[
m = (\xi k)^{-1}
\]

where \( n_2 \) is the power law exponent in the constitutive relationship of the substrate.

In the substrate the stresses and velocities must be bounded as \( z \to \infty \); therefore \( B_1 \) and \( B_2 \) vanish. The perturbing velocity and stress components are

\[
\dot{w} = [B_3 \cos(1/kz) + B_4 \sin(1/kz)]e^{\alpha'kz} \cos(kx)
\]

\[
\dot{u} = -[(\alpha' B_3 + \beta B_4) \cos(1/kz) + (\alpha'' B_4 - \beta B_3) \sin(1/kz)]e^{\alpha''kz} \sin(kx) \tag{A6}
\]

\[
\sigma_{zz} = \tau_1 k/2(\Gamma_1 B_3 + \Gamma_2 B_4) \cos(1/kz)
\]

\[
+ (\Gamma_1 B_4 - \Gamma_2 B_3) \sin(1/kz)]e^{\alpha''kz} \cos(kx) \tag{A7}
\]

where

\[
\Gamma_1 = \alpha''[(4/n_2 - 1) - (\alpha'^2 - 3\beta^2) - m(1 + \alpha'^2 - \beta^2)]
\]

\[
\Gamma_2 = \beta[(4/n_2 - 1) + (\beta^2 - 3\alpha'^2) - 2\alpha'm] \tag{A8}
\]

The equations for the velocities and stresses in the overlying layer can be obtained by replacing \( \tau_1 \) by \( \tau_0 \) and letting
The manuscript.

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Using the method of static condensation described by Zuber at the layer-substrate interface. The solution is determined that normal stress is continuous and shear stress vanishes at compressional instability $q$ given in (1) under the conditions that $n_1$ is the power law exponent in the constitutive relationship of the layer. In this case none of the coefficients vanish; so the general expressions for the perturbing velocities and stresses are

$$\dot{w} = \left[ (A_1 \cos (\beta kz) + A_2 \sin (\beta kz) \right] e^{\alpha'kz}$$

$$+ [A_3 \cos (\beta kz) + A_4 \sin (\beta kz)] e^{\alpha''kz} \cos (kx)$$

(A10a)

$$\tilde{u} = -\left[ (\alpha' A_1 + \beta A_2) \cos (\beta kz)$$

$$+ (\alpha' A_2 - \beta A_1) \sin (\beta kz) \right] e^{\alpha'kz}$$

$$+ \left[ (\alpha'' A_3 + \beta A_4) \cos (\beta kz) + (\alpha'' A_4 - \beta A_3) \sin (\beta kz) \right] e^{\alpha''kz}$$

$$\cos (kx)$$

(A10b)

$$\tilde{\sigma}_{zz} = \tau_0 \left[ A_1 \cos (\beta kz) + A_2 \sin (\beta kz) \right] e^{\alpha'kz}$$

$$+ \left[ A_3 \cos (\beta kz) + A_4 \sin (\beta kz) \right] e^{\alpha''kz} \cos (kx)$$

(A10c)

$$\tilde{\sigma}_{xz} = -\tau_0 k/2 \left[ (1 + \alpha'^2 - \beta^2) A_1 + 2 \alpha' \beta A_2 \right] \cos (\beta kz)$$

$$- \left[ (2 \alpha' A_1 - (1 + \alpha'^2 - \beta^2) A_2) \sin (\beta kz) e^{\alpha'kz}$$

$$+ \left[ (1 + \alpha'^2 - \beta^2) A_3 + 2 \alpha'' \beta A_4 \right] \cos (\beta kz)$$

$$- (2 \alpha'' A_3 - (1 + \alpha'^2 - \beta^2) A_4) \sin (\beta kz) e^{\alpha''kz} \right]$$

$$\cos (kx)$$

(A10d)

We simultaneously solve for the coefficients in the substrate ($B_{1,4}$) and layer ($A_{1,4}$) and the growth rate of the compressional instability $q$ given in (1) under the conditions that normal stress is continuous and shear stress vanishes at the free surface, and stresses and velocities are continuous at the layer-substrate interface. The solution is determined using the method of static condensation described by Zuber et al. [1986].

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