A continuum approach to the development of normal faults

Gregory A. Neumann & Maria T. Zuber
Department of Earth and Planetary Sciences, Johns Hopkins University, Baltimore, Md., USA

ABSTRACT: We present a mechanism for the development of normal faults based on continuum flow in brittle-plastic and ductile rheology. A rate-dependent effect, as observed in rock friction studies, leads to strain localization. Analytical solutions in the wavenumber domain for infinitesimal perturbation theory are used to elucidate the nature of uniform extension of a brittle-plastic medium. Finite element models are then used to investigate strain localization under finite extension. We find that the fundamental mode of extensional instability controls the width of rift-like deformation zones, while higher-order wavenumbers produce secondary faulting on the rift floor. These higher order modes arise naturally for a sufficiently high thermal gradient and for a sufficient fraction of velocity-weakening. The superposition of these higher order wave numbers on the fundamental mode provides a simple physical description for the nucleation of fault structures with regular length scales as a function of the thickness of the brittle lithospheric layer.

1 INTRODUCTION

Linear, macro-scale tectonic features arise from brittle deformation of lithosphere, and usually scale to the depth of the brittle-ductile transition(s). Images of the surfaces of many solar system objects, such as dramatic views of Venus provided by the Magellan spacecraft, illustrate that the development of fault-like features is not limited to Earth. In this study, we investigate the relationship between observed extensional surface features to the interaction of friction and flow laws in the lithosphere.

Near-surface faults result from seismic and aseismic slip along surfaces, with brittle deformation occurring when stresses exceed a limiting strength. At higher temperatures, where ductile flow occurs at lower stress, a temperature-activated power law relates stress and bulk strain rate (e.g., Brace and Kohlstedt, 1980). Quantitative models of extension of crust and mantle based on experimentally-obtained flow laws for quartz, diabase, and olivine aid our understanding of surface morphology (e.g., Zuber et al., 1986; Buck, 1991).

A transition from unstable sliding to stable creep is observed at increasing temperatures and pressures in crustal materials, corresponding to a limiting depth of seismicity. Friction on rock surfaces shows a dependence on strain and loading history (Dieterich, 1979; Ruina, 1983; Linker and Dieterich, 1992) as well as steady-state velocity, that can be described by coupled time-dependent parameters. Study of such "rate and state" variables in materials such as granite shows that the stability of sliding depends increasingly on bulk flow as the brittle-plastic transition is approached (Scholz, 1990). Partly due to limited experimental data and partly due to complexity, few models have incorporated the non-linear interaction of frictional constitutive laws and flow laws. Synthesis of these laws into a working description of the behavior of material at failure in actual geological settings is required to answer such outstanding questions as:

What are the primary controls on initiation and spacing of faults?

How does rheology affect the subsequent development and propagation of faults?

One simple approximation treats the lithosphere as uniform in strength, floating on an inviscid fluid layer. Within such a strong layer, displacement along a fault proceeds via flexure until extensional stresses exceed the strength of intact lithosphere, and a new fault is started (Vening Meinesz, 1950; Weissel and Karner, 1989; Forsyth, 1992; Shaw and Lin, 1993). Another approach treats the lithosphere as a nearly cohesionless plastic medium that yields where a strength envelope is exceeded. Shear stress along the base of the lithosphere controls the growth of instabilities that eventually give rise to faults. Continuum methods that treat the extending lithosphere as a non-Newtonian, viscous fluid have successfully described the characteristic spacing of basin-and-range topography (Fletcher and Hallet, 1983), rift valley morphology (Zuber and Parmentier, 1986), dynamic topography (Chen and Morgan,
Frictional stress-strain relationships in one dimension are, to first order, identical to the rheology of a perfectly plastic material: arbitrary displacements occur above a critical stress $c_\tau$ while only elastic displacement or creep occurs at lower stress. Microstructural observations of brittle-plastic flow at mid-crustal pressures and temperatures reveal a range of deformation mechanisms at the brittle-ductile transition, from localized microcracking to distributed plasticity (Evans et al., 1990; Hirth and Tullis, 1994), for which the stress-strain relationships can exhibit work-hardening, leading to homogeneous strain, or work-softening, leading to localization of strain into faults. Once inelastic deformation begins, its dynamics may be controlled by second-order dependence of friction laws on strain history.

As a step toward merging continuum behavior with friction, we model the lithosphere and asthenosphere as non-Newtonian, incompressible fluids, with isotropic material properties determined by a strength envelope. The coefficient of internal friction can vary in our model depending on strain rate.

In many studies, the location and the angle of faults is prescribed (e.g., Melosh and Raefsky, 1981; Melosh and Williams, 1989) or controlled by the relative importance of magmatism and extension (Shaw and Lin, 1993). In our approach the spacing, and to some extent the dip, of normal faults is controlled by instabilities in non-linear rheology. Finite amounts of extension are accommodated by localization of plastic strain in quasi-linear regions dipping at moderate angles below the surface. The continuum approach reveals an aspect of the faulting process not usually treated, nucleation of faults in the brittle zone and the diffusion of strain on faults as they approach the brittle-ductile transition. We begin by examining the ideally plastic case, as described by infinitesimal necking instability theory. We then adopt more realistic and spatially variable rheologies using finite elements, finding that faults may arise from long-wavelength perturbations in lithospheric thickness.

2 PERFECTLY PLASTIC RHEOLOGY UNDER EXTENSION

Non-Newtonian, temperature-dependent rheology can be described by a power law (Kirby, 1983)

$$\dot{\varepsilon} = A (\sigma_1 - \sigma_3)^n \exp(-Q/RT)$$

where $\dot{\varepsilon}$ is the maximum principal shear strain rate, $\sigma_1 - \sigma_3$ is the principal stress difference, $n$ is the power-law exponent, $Q$ is the molar activation energy, $R$ is the gas constant, $T$ is temperature, and $A$ is a material strength constant. Necking instabilities may develop in a non-Newtonian strong layer under extension (Smith, 1977). A power-law rheology with large $n$ develops pinch-and-swell boudinage with a characteristic wavelength $\lambda$ equal to 4 times the layer thickness. The growth of perturbations to uniform flow in a power-law rheology has been described analytically (Fletcher, 1974; Fletcher and Hallet, 1983; Zuber et al., 1986). Such analysis approximates the perfectly plastic behavior of a cohesionless Coulomb material under uniform confining pressure in the limiting case of
very high \( n \). For a plastic material over a ductile substrate (Figure 1), a harmonic perturbation of one interface at a wavenumber \( k = 2\pi h/\lambda \), can be described by a sum of eigenmodes of the perturbation flow (Zuber and Parmentier, 1986; Ricard and Froidevaux, 1986; Bassi and Bonnin, 1988; Martinod and Davy, 1992). An unstable eigenmode consists of perturbations whose amplitudes grow with extension \( \xi_{\alpha} \) by a factor \( \exp\left( \xi_{\alpha}(k-1) \right) \) given by a growth spectrum \( q(k) \) (Figure 2). A finite amount of extension causes exponential growth of the mode associated with positive \( q(k) \), while modes with negative \( q(k) \) decay. The most rapid growth occurs at a wavenumber \( k = \pi/2, \lambda = 4h \). Other maxima occur at \( 3\pi/2, 5\pi/2 \), etc. corresponding to higher order eigenmodes, wherein several layers of necking and buckling alternate. The behavior of any given perturbation under extension is found by summing the amplified contributions of each eigenmode at all wavenumbers. The growth spectrum decreases in the presence of gravitational restoring forces given by the dimensionless ratio of lithostatic stress to extensional stress, \( S = \rho gh/\sigma_{xx} \). For a medium with a large density contrast at the surface \( S=3 \), the peaks in the growth spectrum are somewhat smaller, and shift to a 25-30% longer fundamental wavelength. Gravity favors normal faults at angles lower than 45\(^\circ\), giving rise to longer wavelengths (Forsyth, 1992). Observations often show the selection of a fundamental wavelength of instability, but the higher order modes also play an important role in the localization of strain. An initial, Gaussian-shaped perturbation to the base of the plastic layer \( z(x) = \exp\left(-x/\delta^2\right) \) of width \( \delta = 0.1h \) may be expanded as the sum of its Fourier components at discrete wavenumbers, and the flow due to each eigenmode calculated after extension (Figure 3). Each component of the Gaussian contributes to the initial perturbation, but only eigenmodes at the odd harmonics of the fundamental wavelength are strongly amplified so as to interfere constructively, resulting in a box-car like disturbance at the surface. The initial perturbation creates a narrow shear zone whose width is controlled by the width of the perturbation, and in the limit \( \delta = 0 \) consists of thin sliplines. The central V-shaped graben is surrounded by horsts. This style of deformation represents a small perturbation to a basic state of extension that resembles, but does not fully describe, the evolution of necking in a plastic medium (Lin and Parmentier, 1990).

The growth factors for \( n=100 \) (Figure 2, dashed line) decrease at higher wavenumbers. In a perfectly plastic medium \((n=\infty)\) shear stress is limited to a critical value \( \tau_c \) and does not increase with increasing strain rate, while for less than ideal behavior, higher strain rates lead to slightly increased stress. Increased stress at higher local strain rates reduces the growth of intensely sheared, short wavelength features such as sliplines. Shear zones in the semi-brittle, mylonitic regime should therefore tend to broaden. Figure 3 demonstrates that the transition to power-law creep rheology impairs localization. A strong brittle-plastic layer overlying a ductile layer with continuously decreasing strength also shows a decline in magnitude of growth factors at higher harmonics (Zuber et al., 1986; Martinod and Davy, 1992). Thus temperature-dependent ductile flow at the base of a strong layer will also impair localization.

3 ORIGIN OF LOCALIZATION

Regions of rapid strain persist in a cohesionless, plastic medium due to the fact that the effective viscosity \( \eta_{\text{eff}} = \tau_c/(2\xi_{\alpha}) \) depends inversely on strain rate. The higher the strain rate, the weaker the material becomes in a linear, Newtonian sense. Regions of weakness tend to concentrate strain. In nature, such weakness may also reflect a change in material properties. Conversely, regions of greater strength will tend to exclude strain.

Localized weakness may arise from material inhomogeneities, thickness or temperature variations, or a combination thereof. For large strength contrasts and nearly uniform material, such as in the example above, a 5% uniform strain might result in hundredfold growth of inhomogeneities at the dominant wavelength. High growth rates therefore can lead to obvious, periodic structures.

Less than ideally plastic behavior, or a transition in strength over a zone of finite width results in declining growth spectra at higher wavenumbers. In such a case, the amplification of a broad range of wavelengths under extension leads to an irregular texture around an overall dominant spacing, while finer scale instabilities decay relative to the fundamental wavelength. Complete localization therefore does not arise under reasonable
assumptions of initial conditions and rheology. Production of large fault surfaces under extension would seem to require more than perfectly plastic behavior. Such behavior, on a fine scale, occurs through an increasing concentration of cracks, leading to catastrophic failure, while subsequent deformation is accommodated via sliding between competent regions. The details of frictional behavior of moving surfaces that provide a mechanism for growth of finite amounts of localized deformation must therefore be explored numerically.

4 FINITE ELEMENT MODELS—APPLICATION TO CONTINENTAL RIFT ZONES

For each viscous finite element, a strength envelope for intact samples is given by Byerlee's law (1978),

\[ \tau_{\text{max}} \leq \sigma_0 + C \rho g z, \]

where \( \rho g z \) represents lithostatic minus hydrostatic stress. The Coulomb strength coefficient \( C \) is related to the angle of internal friction \( \theta \) by \( C = \sin(\theta)/(1+\sin\theta) \). The maximum friction \( \mu = \tan(\theta) \) is about 0.85 - 0.9 and \( C = 0.4 \). Where material undergoes extensive failure, the cohesive strength \( \sigma_0 \) becomes negligible. As \( z \) and \( T \) increase, the strength envelope permits ductile flow to occur at a lower stress. This brittle-ductile transition may not occur at a fixed depth. The local stress field and material properties may evolve with the geometry of instabilities that are not known in advance.

The penalty formulation with reduced integration on quadrilateral finite elements (FE's) is a standard method for incompressible flow (Bathe, 1982; Pelletier et al., 1989). To treat non-Newtonian, temperature-dependent flow, we linearize the constitutive laws for both the brittle and ductile regimes (e.g., Chen and Morgan, 1990; Boutilier and Keen, 1994). The effective Newtonian viscosity \( \eta \) for each FE is computed as

\[ \eta = B \dot{\varepsilon}_2 (1-n)/n \exp(Q/\eta RT), \]

where \( \dot{\varepsilon}_2 \) is the the second invariant of the strain rate tensor and \( B \) is a material strength constant. The resulting maximum principal shear stress is therefore \( \tau = \sqrt{2} \eta \dot{\varepsilon}_2 \). Where \( \tau \) exceeds that allowed by a Coulomb failure criterion the viscosity is given by

\[ \eta = \tau_{\text{max}} / \sqrt{2} \dot{\varepsilon}_2. \]

Thus \( \eta \) is very low near the surface for a given strain rate, while for low strain rates \( \eta \) is large.

We model an extending terrestrial crust composed of a 30-km-thick diabase layer, whose temperature obeys an error function profile defined by \( T_0 = 0^\circ \text{C}, T_{\infty} = 1350^\circ \text{C}, \) and \( dT/dz = 18^\circ \text{C} \text{ km}^{-1} \) at the surface. Such a model could represent the weak lithosphere in regions such as the East Africa Rift system (e.g., Ebinger et al., 1989). Figure 4 shows a FE model with 0.5 km nodal spacing, whose half-width is 25 km. The bottom boundary is shear-free, i.e. no mantle interaction. Boundary conditions are symmetric at the center (no vertical shear stress, no horizontal displacement) and uniform extension at the edges (no vertical shear stress), essentially periodic, at a basic strain rate \( \dot{\varepsilon}_2 = 10^{-14} \text{ s}^{-1} \). A strength envelope for the crust is shown in Figure 5a. The ductile strength is computed using the flow law of Shelton and Tullis (1981) for diabase, with \( \rho = 2700 \text{ kg m}^{-3}, n = 3.4, A = 100 \text{ MPa}^{-n}, \) and \( Q = 260 \text{ kJ mol}^{-1} \).

![Figure 4. Finite element models based on a Byerlee-law, brittle-plastic strength envelope over a power-law, temperature-dependent ductile substrate. Elements on the left of axis of symmetry are shaded according to strain rate, on the right according to stress. Elements at failure are darkly shaded. a) Prescribed extension of the models by 5%, in the presence of gravitational loads, produces a broad, rift-like zone. b) Modified strength envelope incorporating log-strain rate weakening. Model generates regions of high strain rate resembling normal faults.](image-url)
A 0.5-km-amplitude, sinusoidal perturbation applied to the model geometry and temperature field, thins the brittle lithosphere in the center. A strong, brittle-plastic, 12.5-km-thick layer is unstable in extension at \( \lambda = 50 \) km (Figure 2) and undergoes necking. Stresses increase with depth toward the brittle-ductile transition and then decline. Strain rates are highest near the base of the necking region, or rift, while the brittle rift flanks and the plug below the rift undergo little deformation.

Before extending the model, we iteratively recalculate \( \dot{\varepsilon}_2 \) and \( \eta \). Large differences in strain rates and viscosities diffuse slowly throughout the model. Over-relaxation of the change in strain rate by factors close to 2 (Parrish, 1973) accelerates convergence. Having converged to an instantaneous velocity field, we extend the model by increments of 0.2% strain, integrating velocity to second-order. Strain localizes to a 20-km wide region as a consequence of a steep drop in strain rate of the brittle portion of the lithosphere.

5 MODIFICATION TO BRITTLE-PLASTIC RHEOLOGY

The Byerlee's law strength envelope is a linear fit to experimental data for a variety of intact rock samples stressed to failure at high strain rates under a range of confining pressures. Somewhat lower coefficients of friction are measured on creeping surfaces. Fault surfaces exhibit velocity weakening at room temperature and low slip rates in granite and quartz sandstone (e.g., Tullis and Weeks, 1986) while at velocities greater than 10^{-3} m s^{-1} this phenomenon is less pronounced (Kilgore et al., 1993). Such weakening, variously attributed to subcritical crack growth, crystal plasticity, or the time-dependent behavior of asperities, is an important factor in the instability of stick-slip motion on earthquake faults (Brace and Byerlee, 1966). The degree of weakening is roughly linearly dependent on the logarithm of sliding velocity \( V \), being a several percent decrease in friction for a tenfold increase in \( V \) (Scholz, 1990). At temperatures above 300°C, this weakening disappears or becomes velocity-strengthening (Stesky, 1978).

We assume velocity-weakening throughout the brittle regime, with the coefficient of friction \( C \) in equation (2) depending on strain rate as \( C = C_0 - C' \log_{10} (\dot{\varepsilon}_2 / \dot{\varepsilon}_E) \). \( C \) may not exceed 0.4, corresponding to \( \mu = 0.9 \) at a reference strain rate \( \dot{\varepsilon}_E = 10^{-16} \) s^{-1}. For the purpose of illustration, \( C' \) is 10% of \( C_0 \).

Figure 4b shows the stress and strain maps that result from the same initial conditions as in Figure 4a. The diffuse necking zone becomes a single block shearing along sliplines. The localization arises out of the flow law and not from a narrow prescribed perturbation. Strain rates are enhanced in ductile elements at the base of thickened crust, which therefore have reduced effective viscosity. Strain is guided toward lower viscosity, leading to further reduction in strength. This feedback eventually results in highly localized deformation, i.e., the material breaks along plastic sliplines, initially dipping at less than 45°. Deformation then widens and becomes less localized entering the ductile regime, with extension accomodated by a zone of highly extended, flattened elements at depth rather than by tilting and listric faulting.

In contrast to the constant-friction model, higher-order modes of deformation arise at depths of 2 and 5 km, resulting in filling of the rift by a pair of shallower en-echelon faults. The spacing of the faults is controlled by the thickness of the brittle-plastic layer. The generation of higher modes from a fundamental wavelength is not predicted by infinitesimal perturbation theory. Higher wavenumber modes that might arise from numerical instability would be damped out in a relative sense by the continuous transition to ductile flow at low stress from 12.5 to 30 km. The distance \( \zeta \) over which the stress decays by 1/e is about 1.7 km, 0.13 times the thickness \( h \) of the brittle-plastic layer. Zuber et al. (1986) found that for \( \alpha = \zeta / h = 0.1 \), only the fundamental wavenumber is unstable.

![Figure 5. Byerlee's law strength envelopes for planetary lithospheres.](image-url)
6 VENUSIAN APPLICATION – BETA REGIO

As a planetary application of our approach, we consider Beta Regio on Venus, a locus of lithospheric extension interpreted as a rift zone (Masursky et al., 1980). Arecibo (Campbell et al., 1984; Senske et al., 1991) and Magellan (Solomon et al., 1992) data show the region to consist of rift-like topography, with steep bounding faults at the edge of the rift walls and linear fracturing of the rift floor. Such morphology could arise from the velocity-weakening brittle extension described above. However, surface temperature is much higher than that of Earth, and a conventional crust would be too weak to support significant topography.

Recent data on the rheology of diabase (Mackwell et al., 1994) suggest that ductile creep occurs at much higher stress and temperature under the dry conditions found on Venus. Using $Q = 482$ kJ/mol, $n=4.3, A=1150$ MPa$^{-n}$, Figure 5b shows a strength envelope for a higher (25°C km$^{-1}$) Venusian thermal gradient. The more gradual decrease in viscosity ($\alpha=0.33$) requires a higher value of $C'=0.15C_0$ to induce localization and produce a rift-like morphology similar to that in Figure 4b.

A multi-layer lithosphere is known from perturbation theory to give rise to multiple wavelengths of deformation, with aspects of both wide-rift and narrow-rift morphology. Figure 6 shows a lithosphere composed of diabase and olivine under Venusian conditions. The crust is thickened in the center of the model so as to replace the stronger mantle portion of the lithosphere. Faults originate due to a harmonic perturbation when $C' > 0.12C_0$.

In Figure 6, brittle-ductile transitions occur in both the crust and upper mantle. Faulting develops at multiple scales, with the width of the narrowest features being less than the thickness of the brittle crust. Longer wavelength features scale to the depth to the base of the upper mantle lithosphere. Integration of strain rate over time to overall strain of 8% produces conjugate patterns of faulting. Some of the shear stress is transmitted by the mantle to the brittle crust, giving rise to a fault dipping away from the center of the rift. This model suggests the preferential growth of higher-order perturbation modes.

The growth of fault-like zones indicates a mechanism for the development of linear features at multiple scales due to extension. The multiple spacings of parallel linear features ranging from 10 to more than 70 km seen in Beta Regio (e.g., Zuber and Parmentier, 1990) makes it plausible that a model with multiple brittle layers could produce the observed features in an extensional episode.

7 DISCUSSION

Figure 6 indicates a more realistic simulation of the behavior of layered brittle-ductile rheology than previous continuum models. Other extensions to Byerlee’s law have been proposed to account for the differing strength of deforming versus intact materials. The empirical Hoek-Brown strength envelope (Hoek and Brown, 1980) includes parameters for variable amounts of cohesion and interaction of intact masses, and provides a good criterion for surficial faulting in jointed rock masses. At the depths where brittle-ductile interaction becomes important, however, this strength envelope mainly depends on the intrinsic compressive strength of the rock, which is a constant. Attempts to generate sufficient feedback to enhance localization using this law have been unsuccessful.

Simulations where strength envelope varies as a prescribed function of total strain (e.g., Peltzer and Tapponier, 1988; Schultz-Ela et al., 1993) require temporal parameters that are not well-constrained by experiment. The basis of rate-and-state theory is that material strengthens with deformation but weakens over long time-scales. Thus strain-weakening is an indirect consequence of this theory but is not required to maintain localized deformation.

Our approach is mainly for the modeling of fault initiation. Subsequent behavior of faults may also involve flexural rotation. Kinematic rotation of fault blocks is not always observed in extensional settings, and our model provides a mechanism for extension along detachments that does not require listric faulting.

The existence of a low-temperature zone of unstable sliding has been inferred from shallow earthquakes and deformation fabric in narrow shear zones (Scholz, 1988). The minimum width of a fault-like
phenomena tends to damp non-linear interactions at smaller scales that might occur in a refined simulation. Rate-and-state data are available for only a few crustal materials and of these, some exhibit velocity-strengthening at low temperatures (e.g., Reinen et al., 1991). A quantitative description of the strength of faults and of their localization will require further experimental work as well as detailed study of crustal structure.

REFERENCES


